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QUALITATIVE REPRESENTATION AND REASONING FOR SPATIAL AND SPATIO-TEMPORAL SYSTEMS

By

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BAHER AMIN HASSAN EL-GERESY

Jan 2004

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Abstract

This thesis is an in-depth study in the area of Qualitative Spatial Representation and Reasoning. The study is motivated by the potential advantages to be gained from the utilisation of qualitative representation and reasoning techniques in large spatial systems. Qualitative handling of spatial objects and relations has been an active research area in the past 10 to 15 years. The complexity of the issues to be considered has hampered the utilisation of research results in the current generation of spatial information systems and databases. Towards improving this situation, this thesis starts by identifying the main challenges facing the domain of QSRR, namely, the trade-off between expressiveness and efficiency and the trade-off between the completeness and soundness of the approaches. Towards facing the first challenge, a representation formalism is proposed for spatial objects of arbitrary complexity and for different types of spatial relationships between them. Based on the representation methodology, a reasoning formalism is developed to derive the composition of spatial relationships between those objects. The method have been validated by a simple prototype reasoning engine which derives the composition tables between different object types in the topological space. Further more, a study of the application of the methods in the spatio-temporal domain and in uncertain qualitative spaces is presented. The methodology used in this thesis guarantees completeness, in the sense that all relationships between the spatial objects considered are covered. It, however, does not guarantee soundness, in the sense of finding only the physically possible set of those relationships. Accordingly, a set of rules representing topological invariants are also identified which are shown to reduce the set of complete relations to the set of sound ones. The main contribution of this thesis is that it presents a step towards the realisation of the practical application of QSRR techniques in spatial information systems.

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Chapter 1

Introduction

Representing and manipulating spatial or geometric relations are of primary importance in many application areas of large spatial databases such as, Computer Aided Design, Manufacture and Process Planning (CAD/CAM/CAPP), Geographic Information Systems (GIS) and medical and biological databases. As a result Spatial Reasoning (SR) find application in diverse areas such as Assembly Planning, Robotics, Constraint Driven Design and Drafting and Machine Selection and Specification. GIS are based on a range of spatial reasoning techniques for manipulating geographic features on one or more data layers, such as in processing spatial join queries, where sets of geographically referenced features are overlaid in the search for regions satisfying particular constraints. Such application domains are characterised by handling very large sets of entities, relationships and constraints and their manipulation usually involve substantial computational costs.

Qualitative Spatial Representation and Reasoning (QSRR) techniques are being developed to complement the traditional quantitative methods in those domains. Many typical problems could benefit from qualitative manipulation when precise geometric information are neither available nor needed. Applications of QSRR include, qualitative spatial scene specification and scene feasibility problems, checking the similarity and consistency of data sets, integrating different spatial sets, and in initial pruning of search spaces in

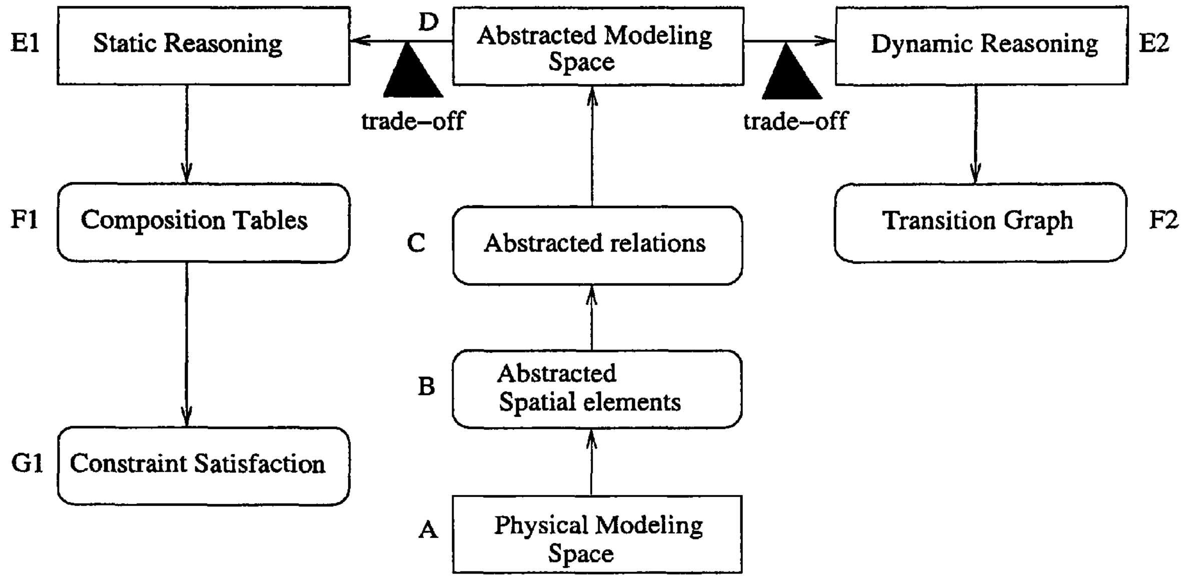


Figure 1.1: Elements of the QSRR process.

spatial query processing. Research is also ongoing for incorporating QSRR in the definition and implementation of spatial query languages. However, the qualitative approach has obvious limitations where useful characteristics of spatial objects such as shape and size are not used. Also, its application becomes limited when exact positions and tolerance constraints are considered. Hence, it can be argued that both the quantitative and qualitative approaches have complementary areas of strength and that any system which can combine the two paradigms in a way which uses their strength would be an effective platform for a range of novel and conventional applications.

Figure 1.1 presents a view of the steps and components of the QSRR process. First, qualitative representation is achieved by selecting and specifying an ontology of objects and relationships in space (steps A-D). Based on the representation scheme, basic reasoning methods can be developed in either of two modes: static reasoning (E_1 and G_1) or dynamic reasoning (E_2 and F_2).

Static reasoning involves the derivation of new relationships ($R_3(x, z)$), given some known

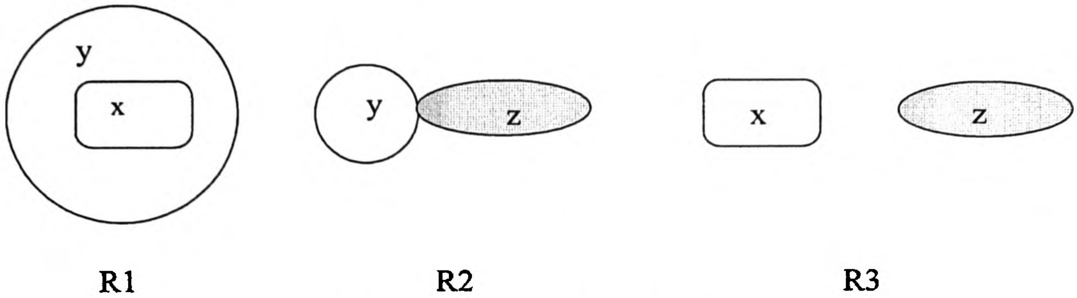


Figure 1.2: Example of static reasoning.

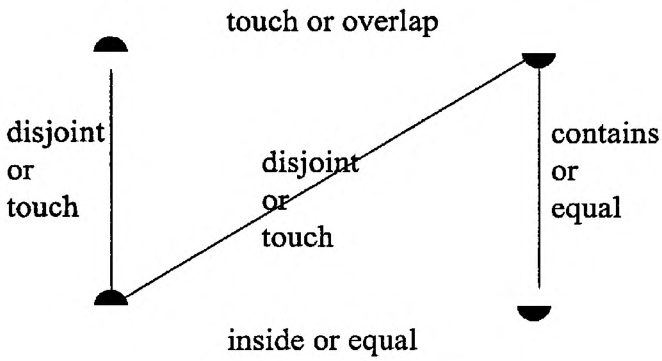
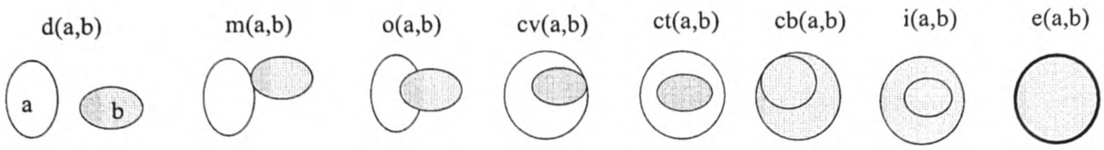


Figure 1.3: Example of a constraint network.

relationships ($R_1(x, y)$ and $R_2(y, z)$). This process is known as spatial relationship composition. For example, given the relationships: *inside*(x, y) and *touch*(y, z), then the relationship *disjoint*(x, z) can be inferred as shown in figure 1.2. If the knowledge of the full set of sound (physically possible) relationships between object x and y are defined, then the full set of combinatorial composition of all relationships can be compiled in what is known as composition (or transitivity) table. Table 1.1 gives the composition table between 3 convex regions [EF91]. Composition tables can be used in the process of constraint satisfaction (G_1 in the figure). Constraint satisfaction in this domain is either the process of checking whether sets of relationships between 3 or more objects are feasible (consistency checking) or in the case of a disjunctive set of relationships between pairs of objects, whether there are subsets of those relationships that are consistent with respect to the composition tables (minimum labeling).



	$d(y,z)$	$m(y,z)$	$i(y,z)$	$cb(y,z)$	$ct(y,z)$	$cv(y,z)$	$o(y,z)$
$d(x,y)$	$noinfo$	$d \vee m \vee i \vee cb \vee o$	$d \vee m \vee i \vee cb \vee o$	$d \vee m \vee i \vee cb \vee o$	d	d	$d \vee m \vee i \vee cb \vee o$
$m(x,y)$	$d \vee m \vee ct \vee cv \vee o$	$d \vee m \vee e \vee cb \vee cv \vee o \vee =$	$i \vee cb \vee o$	$m \vee i \vee cb \vee o$	d	$d \vee m$	$d \vee m \vee i \vee cb \vee o$
$i(x,y)$	d	d	i	i	$noinfo$	$d \vee m \vee i \vee cb \vee o$	$d \vee m \vee i \vee cb \vee o$
$cb(x,y)$	d	$d \vee m$	i	$i \vee cb$	$d \vee m \vee ct \vee cv \vee o$	$d \vee m \vee e \vee cb \vee cv \vee o \vee =$	$d \vee m \vee i \vee cb \vee o$
$ct(x,y)$	$d \vee m \vee ct \vee cv \vee o$	$ct \vee cv$ o	$e \vee i \vee cb \vee ct \vee cv \vee o \vee =$	$ct \vee cv \vee o$	ct	ct	$ct \vee cv \vee o$
$cv(x,y)$	$d \vee m \vee ct \vee cv \vee o$	$m \vee ct \vee cv \vee o \vee$	$i \vee cb \vee o$	$e \vee cv \vee ct \vee o \vee =$	c	$ct \vee cv$	$ct \vee cv$ o
$o(x,y)$	$d \vee m \vee ct \vee cv \vee o$	$d \vee m \vee ct \vee cv \vee o$	$i \vee cb \vee o$	$i \vee cb \vee o$	$d \vee m \vee ct \vee cv \vee o$	$d \vee m \vee ct \vee cv \vee o$	$noinfo$

Table 1.1: Composition table for the eight topological relations between simple regions. ([Egenhofer 91])

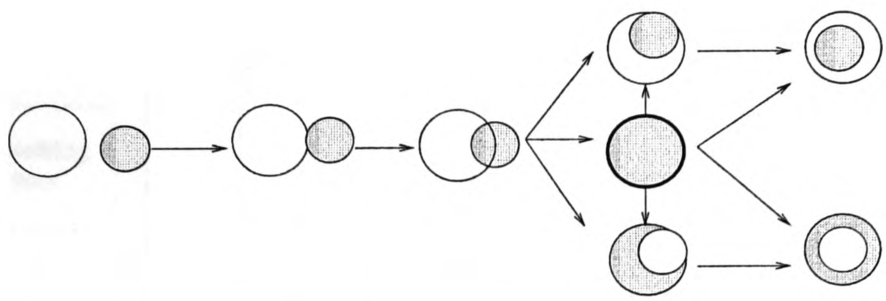


Figure 1.4: Example of a transition graph for two simple regions.

Dynamic reasoning (process E_2 and F_2) is concerned with ordered arrangement (with respect to gradual change) of the full set of possible relationships between two objects, resulting in a "transition graph". In figure 1.4 the transition graph of the eight possible relationships between two simple regions is given.

Development of QSRR techniques faces two major challenges, namely, achieving completeness and soundness and optimising the trade-off between expressiveness and efficiency.

Completeness is defined as the ability of the approach to model all physically possible relations that exist in the physical modeling space. Soundness is the ability of the approach to model physically possible relations only. Expressiveness is the ability of the approach to model different types of objects, spaces, and relation types. Efficiency is related to the reasoning power of the formalism over the modeled relations.

The issues of completeness and soundness needs to be addressed on the representation and the reasoning levels. The aim of the representation process is to map the set of all physically possible scenarios from the physical modeling space A into the abstracted modeling space D, as shown in figure 1.1. If the set in the physical space is $>$ the set in the abstracted space, then the representation may be sound but not complete. On the other hand, if the set in the physical space is $<$ the set in the abstracted space, then the representation could be complete but not sound. Both sets need to be equal for a representation to be both sound and complete, as shown in figure 1.5.

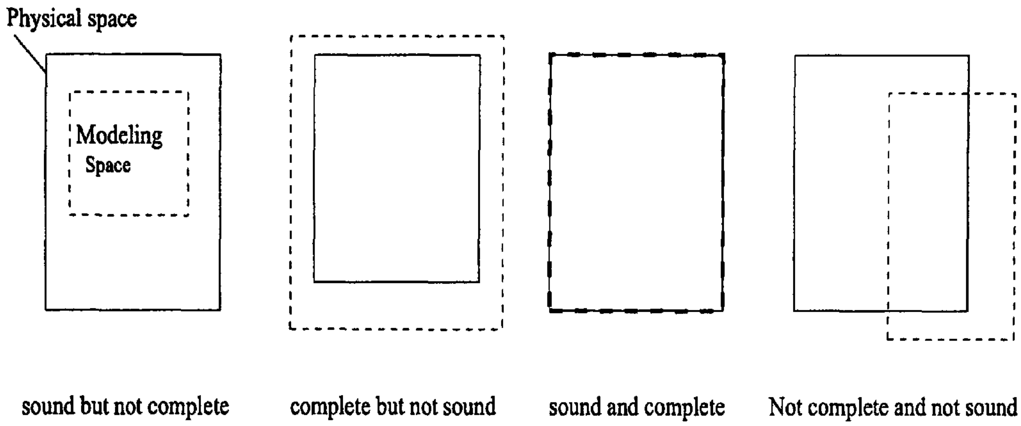


Figure 1.5: Completeness and soundness of qualitative representations.

In the case of static reasoning, completeness and soundness need to be proved for the method used to apply the composition process. Also, in the case of dynamic reasoning, the transition graph has to capture all physically possible transitions between all possible relations correctly to be both sound and complete. Figure 1.6 gives a transition graph for the relation between a line and a region due to Egenhofer [EM95].

The other major challenge noted above is the trade-off between the representation power and reasoning power, or between expressiveness and efficiency [RCC92b]. It is agreed that the more expressive the approach the less efficient it is in terms of reasoning power [LB85], both static and dynamic.

To understand the dimensions of the problem further, let us now have a closer look at the components of figure 1.1. The physical modeling space A is the real world representation of objects and relationships. Figure 1.7 highlights the major challenges to be addressed with respects to handling objects and relationships. The topology, shape and orientation of spatial objects are the main properties which need to be represented. Objects may be spatial or spatio-temporal, i.e. change with time, or just temporal. Relationships in the spatial domain can be classified between three main types, viz, topological (e.g. touch and overlap), proximal (near and far) and directional or orientation (left and north). Also, knowledge in the physical space may be incomplete or uncertain with respect to objects

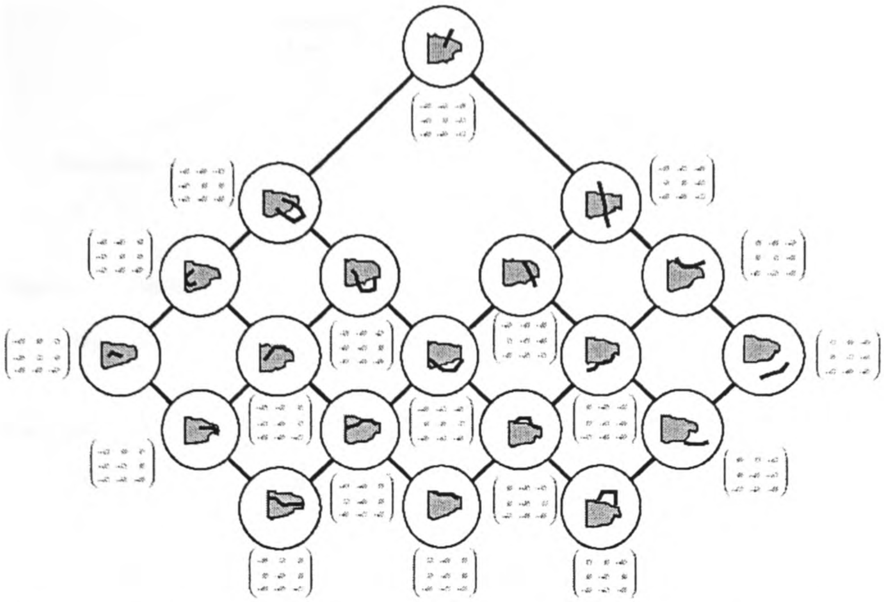


Figure 1.6: Conceptual neighbourhood for line-region relations [EM-95].

or to the relationships between them.

The representation of objects and their embedding space defines basic ontologies which characterises the possible QSRR approaches. Five issues related to the development of such ontologies can be recognised, namely, primitive entities, embedding space, dimensionality, object complexity and object similarity. These issues are summarised in figure 1.8.

Primitive entities: Two approaches can be identified for the choice of primitive entities in space. The region-based approach where objects are represented as wholes, as in [CBGG97]. No distinction is made between the objects' components, e.g. whole regions. The other approach, denoted the point-based approach, is when objects are represented by components of the object itself, e.g. its boundary and interior, as a point-set.

Embedding space: Two ontological choices of the embedding space are between dense (or continuous) spaces and discrete (non-connected) spaces.

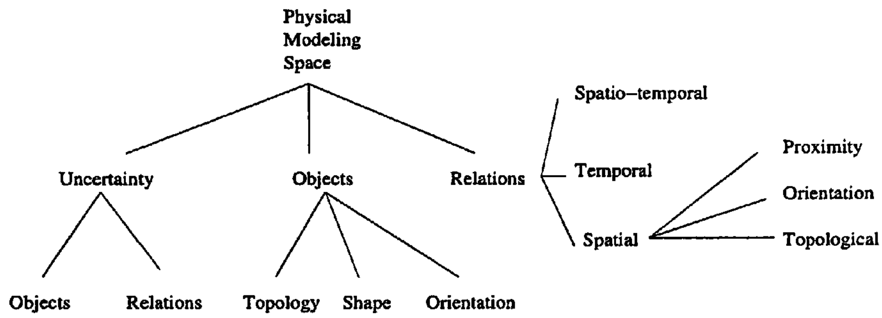


Figure 1.7: Dimensions of issues in physical modeling space.

Dimensionality: This issue is related to the relative dimensions of objects with respect to the embedding space.

Object-Space dimensionality refers to whether the modeled object is represented in a space of equal, higher or lower dimension. An example of equal dimensions, is when a region is represented in 2D space. This is in contrast with the case of regions represented in 3D space. In lower dimension modeling spaces, objects are normally represented by their projections, e.g. regions represented by projections on orthogonal axes (MBR).

Object complexity: This issue is concerned with the ability to handle different levels of object complexity, e.g. simple lines and regions, as well as regions with holes and composite regions, made up of disjoint components.

Object similarity: This issue is concerned with handling objects which may be of different types and dimensions, e.g. by considering the representation of and reasoning over spatial relationships between regions and lines.

Choices made with respect to any of the above ontological issues will have an impact on the efficiency and expressiveness of the approach.

Two approaches can be classified for the representation of spatial relations, namely, set-theoretic approaches (normally associated with point-set representation methods) and

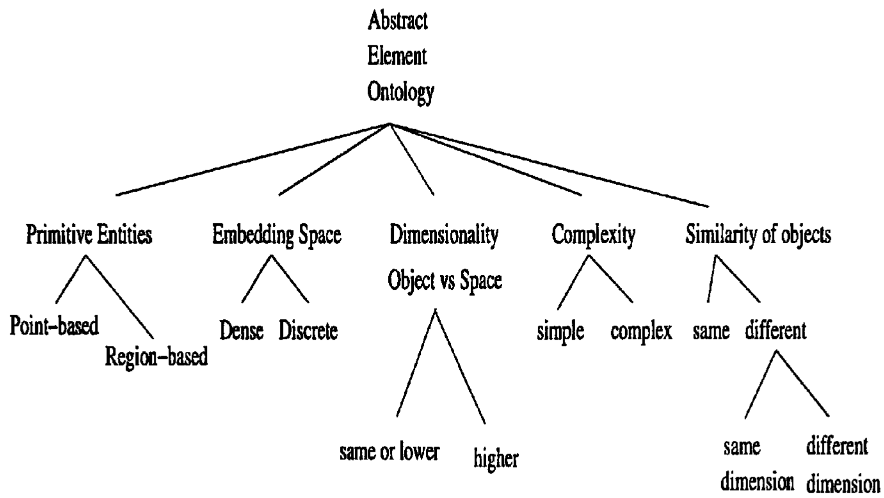


Figure 1.8: Ontological issues for QSRR.

logic-based approaches (normally associated with regin-based representation methods). The two methods can be classified according to the primitive relation or operator they employ, namely, intersection-based and connection-based respectively.

Static reasoning is in essence the process of automatically deriving new, implicit, spatial knowledge, using existing, explicit knowledge. Table 1.1 is a composition table resulting from the application of static reasoning over the full set of possible topological relationships between two simple regions.

Dynamic reasoning is the process of defining all possible transition scenarios between the possible set of spatial relationships between two specific objects. The finer the granularity of the spatial relations represented, the more expressive the methods are, and the harder it is to extract the transition graph. Transition graph must account for all possible transitions (complete) and must not include any non-possible transitions (sound).

Three main approaches for dynamic reasoning can be identified. Firstly, Visual reasoning is when dynamic reasoning is carried out manually. Secondly, simulation algorithm may be developed which gradually alters the size of the object and its relative position to deduce the graph. Finally, a formal model of continuity and transition may be developed for the objects considered.

This thesis attempts to approach the two main challenges in the domain of QSRR in spatial and spatio-temporal domains, namely, the expressiveness-efficiency challenge and completeness-soundness challenge.

From the discussion above, it can be seen that the expressiveness issue is related to the physical modeling space and the ontological considerations. The more types of objects, object characteristics and relations the formalism can represent, the more expressive it is. On the other hand, the more flexible the ontology is, i.e. representing different dimensions of spaces and objects, non-similar and complex objects, the more expressive the formalism will be. Hence, expressiveness can be described by its breadth (covering more the physical modeling space types) and its depth (expressing more different and complex objects). The efficiency of the two types of reasoning power, namely, static and dynamic reasoning, shall be considered as well.

Completeness and soundness should be measured with respect to both the representation and reasoning methods. Approaches to QSRR normally strive to achieve one of either of the two criteria of completeness and soundness. Hence, approaches can be categorised in this respect as completeness-first approaches and soundness-first approaches. Soundness-first approaches are usually based on holistic methods of object and space representation. The identification of all the possible relations between objects in a particular domain is left either to a process of visual reasoning or to some quantitative method utilised to approach completeness. Completeness-first approaches usually adopt the compositional method of representation where elements of objects and space are identified followed by the definition of spatial relationships over those elements. Soundness-first approaches could not guarantee completeness, except possibly for very simple configurations. On the other hand, completeness-first approaches may produce non-valid configurations which may be excluded by the application of soundness rules. In this work the breadth and depth of the effectiveness consideration related to representation shall be addressed.

1.1 Objectives

The objectives of this thesis are to address the following main challenges, namely,

1. *Effectiveness (expressiveness) - Efficiency (reasoning power)* trade-off: the aim is to work towards fulfilling both sides of the trade-off.

2. *Completeness - Soundness* challenge: the aim is to achieve both completeness and soundness for both representation and reasoning in space.

   In addition, a further goal of the thesis is,

3. To consider *Generality* in terms of breadth and depth of expressiveness by developing a formalism that address many aspects of the physical modeling space as defined in figure 1.7 and also deals with objects of arbitrary complexity and different types as shown in figure 1.8.

1.2 Methodology

The work in this thesis extends and substantially modifies the point-based intersection-based approach, proposed originally by Egenhofer et al [EH90]. Objects of arbitrary complexity are studied by considering adjacency of object and space components. The components are not restricted to three components; boundary, interior and exterior as defined in [EH90]. A new approach to static reasoning is developed based on the proposed representation methodology to reason over complex objects of arbitrary complexity, and hence, addressing the depth aspect of the expressiveness issue as well as the efficiency issue.

The method will be extended to address the breadth of expressiveness without losing its efficiency power by considering different types of spatial relations, namely, orientation and proximity relationships. Also, issues of uncertainty of object representation as well as representation in space and time shall be considered.

The approach proposed in the thesis is essentially a completeness-driven approach. Soundness of the method shall be studied and rules for deriving sets of physically plausible relations will be investigated.

1.3 Thesis Structure

The thesis is organised as follows. Chapter 2 presents a review of related work on the various aspects of qualitative representation and reasoning within the domain of this thesis. In chapter 3 the representation and reasoning formalism are developed for the topological domain and applied over objects of arbitrary complexity. In chapter 4 extensions and applications of the formalism are defined for the orientation, cardinal direction and proximity domains. Chapter 5 will show how the method is extended to deal with representation and reasoning in the spatio-temporal domain. In chapter 6 the reasoning formalism is further modified to deal with incomplete knowledge and to reason in the absence of composition tables. In chapter 7, the issue of uncertainty shall be explored and possible ways of handling this issue are proposed for both the representation and reasoning formalisms. Chapter 8 presents a set of general rules to achieve soundness of representation based on topological invariants. A discussion, and conclusions from the work are drawn in chapter 9 and a view on the future is given. A prototype spatial reasoning engine which applies the formalism proposed is described in the appendix.

Chapter 2

Literature Review

Related work on representation and reasoning of qualitative space can be described and classified from different points of views. Firstly, a classification can be made using issues related to the physical modeling space, namely, topology, orientation, space-time and uncertainty. Secondly, approaches can be described according to their ontological aspects. Since ontological choices affects the types of physical modeling space; in this review, the ontological choices are first used to classify related work, and then reflections are made on the different modeling spaces.

From an ontological point of view, approaches are generally those whose representation or abstracted modeling space dimension is equal to or greater than the objects' dimension, and those whose abstracted modeling space dimension is lower than the objects' dimensions. Approaches based on a lower dimension modeling space imply that they use the projection of the object rather than the object itself, i.e. objects are represented by their minimum bounding rectangles or cubes. Such approaches makes it difficult to deal with complex objects as a whole and are prone to errors when considering the representation of relationships. However, these approaches benefit from simplicity and exploitation of well developed temporal algebras, originally developed by Allen [All83]. On the other hand approaches which model objects in a dimension equivalent to, or higher than their physical space define their primitive entities as a point (point set) or a region.

Hence, three main categories of approaches can be identified as follow,

- Approaches that define an abstract modeling space dimension less than the objects' dimensions, (lower modeling dimension).
- Approaches that define an abstract modeling space dimension greater than or equal to the objects' dimensions, either,
  - using points as primitive entities (point-based), or,
  - using regions as primitive entities, (region-based).

Lower modeling space dimension or (projection-based) approaches: In these approaches, objects are represented by their MBR, and projected on the axes or plane. Relation between objects are defined by the relations between their projection. Examples of these approaches are found in [PS94, PTSE95, Tra98]. A drawback of this approach are the errors that may occur when defining topological relationships. Since, it may be possible for projections of objects to intersect while the actual objects are not intersecting, false interpretation of connectivity relationships may be inferred. Also, the representation of complex objects is problematic when considering the projection of possibly disjoint components. Those approaches benefit, however, from simplicity and possible exploitation of well developed temporal algebras, originally proposed by Allen [All83].

Point-based (intersection-based) approaches: In these approaches, point sets are used as the main primitive entity. Those approaches are generally based on the work of Egenhofer et al [FE92, PE88], where objects are represented by their boundary, interior and exterior. Topological relations are represented by a 9 intersection matrix which store the intersection of each of the 3 components of one object with the components of the other objects. The intersections are considered to be either empty or non-empty. Examples of this approach are found in [ECP94, Ege93].

Region-based approaches (connection-based): Region-based approaches are based on the region-connection-calculus (RCC), developed for convex regions by Cohn et al [CRCB93, CCR93, CV99]. The calculus is based on Clarke's calculus of individuals [Cla81]. A region is used as the main primitive entity and *Connect* is used as the main primitive relation with which more specific relations can be defined. Originally, it was developed to define the eight relations between two convex regions as was defined with the point-based approaches above.

In what follows, different approaches for the physical modeling space are reviewed according to the different types of space considered and cross-referenced with the ontological classifications defined above.

2.1 On Representing Topological Relationships

Two main approaches for representing and reasoning over topological relationships are identified, namely, the region connection calculus (RCC) [RCC92a, CRC94, CRCB93, RC92, CCR93] and the intersection matrix [EFJ89].

The RCC method employs first-order logic to define a spatial logic based on regions and connectedness for the definition of topological, orientation and distance (near, far) relationships. A many sorted first-order logic system LLMA [RCC92a] was used to prototype the spatial logic. The basis of their spatial theory is the calculus of individuals developed by Clarke [Cla81]. The definition of the topological relations is based on a primitive dyadic relation of connection $C(x, y)$ or x *connects with* y. This relation describes the fact that two bodies are connected if they share at least one point in common. Based on this concept a set of eight binary relations were defined to describe different degrees of connectedness between regions as shown in figure 2.1.

Examples of the definition used are,

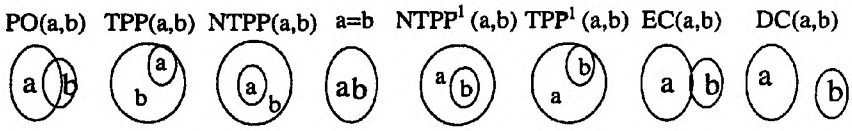


Figure 2.1: Topological spatial relationships defined in [Randell et al 92].

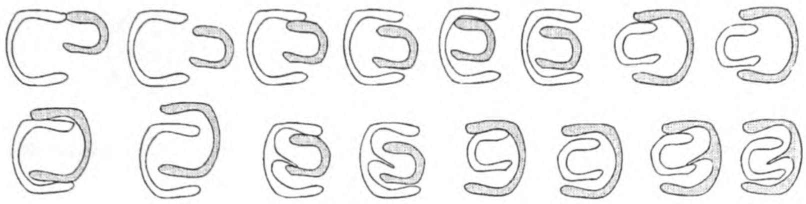


Figure 2.2: Examples of topological relations between concave regions defined in [Cohn et al 93].

- $DC(X,Y) \equiv_{def} \neg C(X,Y)$
- $P(X,Y) \equiv_{def} \forall Z[C(Z,X) \rightarrow C(Z,Y)]$

where $DC(X,Y)$ is read as x is disconnected from y and $P(X,Y)$ is read as x is part of y.

An extension of the above theory to cover more shapes, and thus increasing the depth of its expressiveness is presented in [CRCB93] to express the relationships between concave regions. A new function to represent the convex hull ($conv(x)$) is introduced which is used for the definition of three further relations, `inside`, `partially inside` and `outside`, which resulted in the extension of the eight base relationships into 22 relationships, some of which are shown in figure 2.2. Following the same approach Cohn et al [CRC94] refined the set of relations where the original base set was extended to a set of 31 base relations. This demonstrates a main feature of the soundness-driven approach where the formalism has to be revised and extended when considering some different shapes in order to represent the complexity of the shape. Vieu [Vie93] presented a similar approach to that of Randell et al [RCC92a], which is also based on Clarke's calculus of individuals, but which was mainly formalised for the representation of topological relations in the geographic space. The approach has been demonstrated to be extensible to the representation of regular

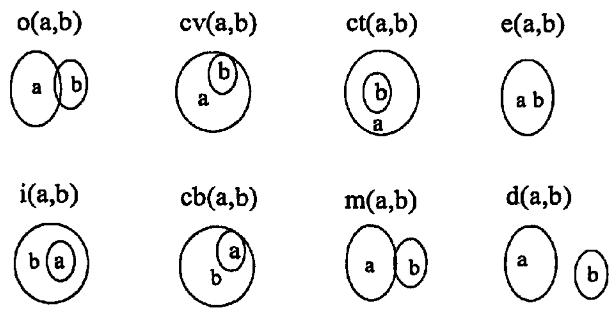


Figure 2.3: The eight topological relations between two simple regions as defined in [Egenhofer 90].

shapes such as squares and circles.

Randell et al [RW01] assumed a region-based ontology of the RCC-8 and extended Galton's line of sight [Gal94] relations into 20 relations in the region occlusion calculus, ROC-20. Finally, Cohn [Coh01] represented cell structure using 47 definitions and 17 axioms, however, his representation did not capture the double helix orientation.

The intersection-based approaches uses objects represented by their boundaries and interiors and an atomic intersection relation between the objects' components which takes a value of true or false. The combinatorial combinations of these intersections covers the set of all relations in the domain.

Egenhofer [EF91] developed a formalism for the representation of regions in two dimensional space. A 2×2 intersection matrix representing the intersection between the boundaries and the interiors of the objects is used. Empty and non-empty intersection sets are used as the atomic relations in this case. The combinatorial possibilities in this matrix result in a set of 16 relations of which only 8 are possible as shown in figure 2.3. Using the boundary and interior decomposition, he also represented the relations between two lines in a one dimensional space [PE88].

To increase the discriminative power of the approach two different methods can be used:

1. increasing the resolution through decomposing the object into more components, or,
2. increasing the precision through defining more constraints on the atomic relations which results in producing more granular atomic relations [Gal98].

To increase the precision of his method, Egenhofer extended the 4-intersection matrix into a 9-intersection one by introducing the exterior of an object as one of the components to be considered in the intersection process between two regions in [Ege91] and between regions in a raster representation [ES93]. He has also used a representation of the boundary of a line as two separate points in [Ege93] with the result that more qualitative relations between the lines were distinguished.

Clementini et al [CDO93] added the dimension of the intersection as another atomic relation, to be considered with the empty non-empty intersection tests in the 4 intersection matrix [EF91]. This atomic relation resulted in three constraints, namely, whether the result of the intersection is a point, a line or an area, and has further increased the precision of the method.

Another extension to the above method was developed by Franzosa & Egenhofer [FE92] where the dimension of the intersection as well as the number of occurrences of the intersection were taken into account. Although this extension increased the discrimination ability of the approach, using the number of intersections as one of the constraints introduces a quantitative aspect to the approach. Towards increasing the resolution of the approach, a further refinement was proposed in [FE92], namely, to consider whether the intersecting component is a 'touching' or an 'intersecting' one by examining the points that proceed and follow the contact point of the two components.

In [ECP94] the intersection-based approach was extended to handle topological relations between regions with holes. Each region is decomposed into simple regions and combinations of the possible relations between components were derived to make the representation complete. Again some constraints were added as rules to eliminate non-feasible and redundant relations from the resulting set. A reasoning method based on 8 rules [Ege91] was proposed for reasoning over regions and was used to produce a composition table similar to that developed in [CRCB93]. Relations between lines [Ege93], and lines and regions [ME94] extended the variety of objects handled. In [ES93] the approach was extended to discrete (raster) space for simple regions. To increase the precision of different relations. Vazirgiannis [Vaz00] defined different degrees of a single qualitative relation between simple regions by considering the ratio between the intersected part of an object over such part, for example, an overlap ratio is defined as the ratio $(A_0 \cap B_0)/B_0$, where A_0 and B_0 are the interior of A and B respectively.

A main limitation with the 9-intersection matrix is its reliance on the definition of objects by three components of interior, boundary and exterior. To represent complex objects, the objects need to be tailored by dividing them up into smaller objects which could fit the representation scheme. For example, in representing relationships between two regions with holes, (A, H_A) and (B, H_B), where H denotes a hole, holes were realised as regions in their own right. The relationship between the two objects was translated to the combinatorial relations between regions and their component holes, i.e. the relationship was represented as a constraint network between 4 regions (A, H_A, B, H_B).

Both of the above approaches represent objects in equal space dimensions. A third approach to represent objects in spaces of lower dimension is the projection-based approach. Due to the simplicity of the relations between two lines in a one dimensional space, many investigators have tried to exploit this property in constructing spatial relations in higher dimension spaces. This is done by "projecting" the object on some axis (or objects in space) with each projection creating a relation in a one dimensional space. Different combinations of different relations in different projections can define the required set of relations.

Guesgen [Gue89] adapted Allen's [All83] approach by using the relations `left-of` in place of $<$ and `right-of` in the place of $>$. By projecting the body onto 3 axes, he suggested that the full set of spatial relations would be recognized in 3D space. The same approach was suggested by Malik and Binford [MB83] in an earlier work. In image processing applications Chang et al [CYDA88] proposed a projection approach called the 2D-string to define spatial relations between objects in an image database. The projection is carried through an object called the *Point of View Object* (PVO). Chang used the object centroid in the projection process and hence the atomic relations $(<, =, >)$ were used between the two centroid points.

Lee and Hsu [LH91] have extended Chang's method in image database applications to achieve expressiveness by considering the projection of a line as opposed to that of a centroid. Each projection line was decomposed into its beginning and end points which gave 4 combinations for each projection (a matrix of 4 elements each having three possible atomic relations). Their combination gave 169 possible relations between the two objects as shown in figures 2.4 and 2.5. In all projection methods, as noted from figure 2.4, each object is represented by its minimum bounding rectangle (MBR). As mentioned earlier, a major problem with MBR approach is the possible wrong interpretation of topological relationships.

Papadias [Pap94] used a mixed Projection-Intersection approach to represent orientation and topological relations symbolically. As mentioned earlier, he used the projection-based approach to define orientation and the intersection-based method of [EF91] to represent topological relations. To represent a traffic scenario, Renz [Ren01] represented the cars and their regions of influence as directed intervals and a road as the underlying line. Twenty six base relations were used in this case.

Observing the analogy between non-convex intervals and a spatial entity composed of non-connected parts, Claramunt [Cla00] extended Ladkin's algebra on non-convex intervals to the union of spatial regions. Based on the concepts of adverbs: mostly, completely, partially, occasionally, entirely and never, the 8 relations between convex regions could be modified to express relations between the union of convex regions which can exist in real geographic space. The advantages of the approach is its favorable proximity to

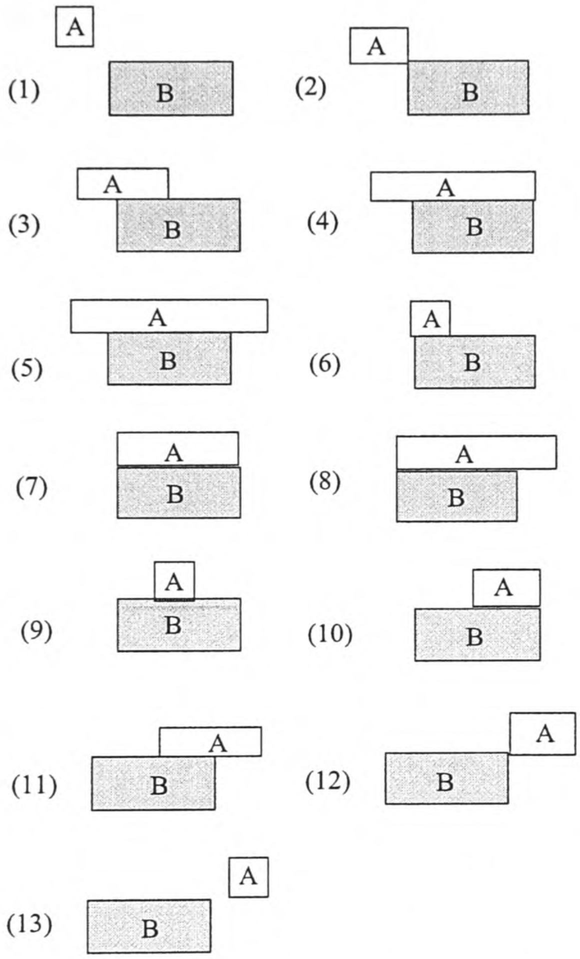


Figure 2.4: The 13 types of spatial relations in one-dimensional space ([Lee & Hsu 92]).

Figure 2.5: The 169 types of spatial relations between two rectangles in two-dimensional space ([Lee & Hsu 92]).

common-sense reasoning.

2.2 On Uncertainty

The study of spatial uncertainty has been addressed in various works in the past. Approaches can generally be categorised under two classes, namely, *Exact* models and *Fuzzy* models. "Exact" approaches are generally based on one of two models of representation, namely, the Region-Connection-Calculus (RCC) [CG94, CH01, RS01, CDF97, CDF96b, CG94] and the intersection-based approach [TN, Zha98, TJ, Sch01]. They deal mainly with simple convex regions and range uncertainty over the boundary of the regions.

The "egg and yolk" approach [CG96] based on region-connection calculus uses an analogy for defining objects, where the difference between the egg and the yolk represents a range of uncertainty of the object's boundary. Different sets of relations have been identified for those objects; 46 relations in [CG96] using first order logic and 44 relations using the intersection-based approach [CDF96a]. Clementini et al [CDF97] added a further set of 12 relations for representing composite objects with indeterminate boundaries.

In [CH01], the changes in the egg and yolk were considered for the purpose of defining a spatio-temporal interpretation of the method, for example, by noting the increase, decrease or the stability of the egg and yolk respectively. Several approaches were presented following the egg-yolk and the RCC-8 methods. In [Ste00] relationships are defined through truth values of 3 axioms: A is part of B, B is part of A, and A is part of the complement of B, which enabled the definition of the RCC-8 relations. By considering a 3 value logic for each of the 3 statements (True, False, Maybe), the relations between vague regions were defined. Roy and Stell [RS01] used three relations as a variation of the "connected" relation, namely, adverbs, such as definitely, possibly and definitely not, together with part-of relations to define relations between objects with undetermined boundaries.

A three-value logic was also used by Erwig [ES97] where the intersection between interiors and boundaries of two objects can take one of three values, 1, 0 or *maybe*.

Traverse [Tra98] proposed a projection-based approach for spatial relations based on interval relations, where the representation of incomplete or vague intervals was based on a combination of relations on each axis.

In the fuzzy approaches, [TN] used an aggregate uncertainty with values of 1.0 and 0.5 concentric regions of core and support to define the set relations as above. In [Zha98], the fuzzy set was divided up into more than 2 concentric regions with values between 1 and 0. Some works have also addressed the definition of fuzzy complex regions [Sch01] and a degree of belief is assigned based on a ratio of representation of the characteristic feature, e.g. the area of overlap to the area of one of the objects. Applications of fuzzy relations has been demonstrated in [TJ] in the domain of guiding autonomous vehicle motion.

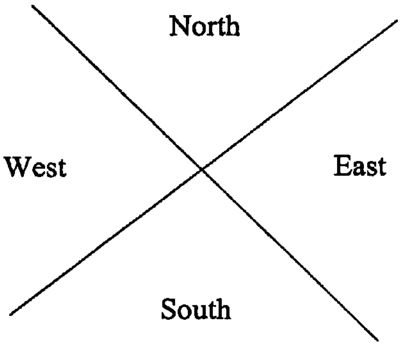
2.3 Orientation and Cardinal Directions

Peuquet and Ci-Xiang [PCX87] proposed a model to represent the directional relationships between polygons of arbitrary shapes and size. They used a set of configurations to depict possible variations of the relation with different sizes and shapes. Different conditions leading to each relation were then recognized. For example, the relationship $east(P_1, P_2)$ can be expressed as $East(P_2, P_1) \leftarrow ((cond1 \cap cond2) \cup (cond3 \cap cond4))$.[1]

Again it was shown how it is necessary to revise the formalism every time a different object shape is considered.

Frank [Fra92] used a rectangular division of space rather than the triangular model used by Peuquet and Ci-Xiang in representing the cardinal direction. He demonstrated some

[1] cond1 : west line of P_2 should intersect some portion of P_1.
cond2 : eastline of P_2 should not intersect any portion of P_1.
cond3 : if the centroid of P_2 falls within the polygon boundary of P_1 then cond2 does not hold.
cond4 : west line of P_2 must not intersect any other part of P_2.



(a)

	N	E	S	W
N	N			
E		E		
S			S	
W				W

(b)

Table 2.1: (a) A level of conical division of space for direction relations and (b) Definite results in their composition table. (Adapted from [Frank 92]).

advantages in reasoning certainty using the modified division. Hernandez [Her91] combined the representation of topological and orientation relations by dividing the space into 32 possible segments to define the relative position of objects. A similar approach for (pictorial) representation of the orientation relations between vectors was proposed by Freksa [Fre92]. In the latter three references, completeness as well as soundness of the representation were approached by dividing the relation space. However, since all the possible relations in the domain are pre-recognized, any increase in the precision or the resolution requires the introduction of new definitions for the possible set of relationships.

Zimmerman [Zim92] presented a qualitative representation of measurements (Δ calculus) which is based upon the comparison of two measures and the difference between them for the orientation and distance relationships. cardinal directions are ordered relations for which compositions can be directly derived. Frank [Fra92] presented the composition table for such relations using a conical and rectangular decomposition of the space. Both tables and the corresponding space decompositions are shown in tables 2.1 and 2.2.

NW	N	NE
W	Oc	E
SW	S	SE

(a)

	N	NE	E	SE	S	SW	W	NW
N	N	N	NE				NW	NW
NE	NE	NE	NE					
E	NE	NE	E	SE	SE			
SE			SE	SE	SE			
S			SE	SE	S	SW	SW	
SW					SW	SW	SW	
W	NW				SW	SW	W	NW
NW	NW						NW	NW

(b)

Table 2.2: (a) A level of rectangular division of space for direction relations and (b) Definite results in their composition table. O_c represents a neutral zone (Adapted from [Frank 92]).

Although order relations can be utilized in reasoning over point-shaped objects, they cannot be directly applied when the actual shapes and proximity of objects are considered. In this case spatial factors such as shape, size, and proximity of the objects disrupt the strict order on which the precise reasoning is based. Peuquet & Xiang [PCX87] discuss these issues and introduce several rules for the definition of cardinal direction relations to handle the effect of these factors. In this case the derivation of the composition table is no longer a systematic process.

The orientation relations (front, back, left) are similar to the cardinal direction relations in that they are defined in terms of semi-infinite areas around an object. Freksa [Fre92] defined a composition table for the orientation relation between two points using the vector between them. Hernandez [Her91] defined the composition table for orientation relations between points. However, transitivity of the order relations depends upon a single frame of reference which facilitates the propagation of the relations. Retz-Schmidt [RS88] recognizes three different frames of reference for the orientation relations. These

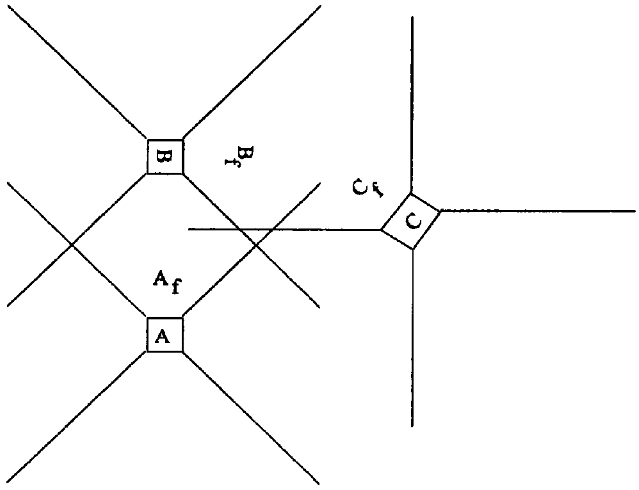


Figure 2.6: Intrinsic orientation relationships. The knowledge of the orientation of object B with respect to A and object C with respect to B does not indicate the relation between objects A and C.

are: a) *Intrinsic:* when the orientation is determined by some inherent property of the frame object (car front), b) *extrinsic:* when external factors impose an orientation on the reference object (direction of car motion), and c) *deictic:* when the orientation is imposed by the point of view from which the reference object is seen.

Orientation relations in the intrinsic frame of reference are not order relations and no direct propagation of the relations is possible. For example, if B is `in-front` of A and C is `in-front` of B as shown in figure 2.6, no information can be inferred between A and C.

Reasoning using different types of relations was considered by Frank [Fra92] where the distance and cardinal direction relations were employed in a query of the form "A is far north from B", "C is near south from B", what is the relation between A and C?. Hernandez [Her91] combined reasoning over topological and orientation relations. Zimmerman [Zim93] combined reasoning over orientation and distance relations using a spatial reference frame similar to that of Freksa and a calculus for representing point-like measures (Δ-calculus).

Efforts were also made to increase the expressiveness on the directional and orientation dimension. In [PWT02] a previous 2D model of orientation relations (6 relations) was extended to the 3D domain. This is considered as increasing the depth of the expressiveness. Towards the same goal, Goyal [Goy00] used a 3 levels model to represent different levels of details of directional relations. Using an intersection matrix approach, the matrix represented the result of intersection of 9 tiles of object A and its embedding space with those of object B. On a coarse level, the matrix captured the binary relations of empty or non-empty set of intersection. On a more detailed level each entry in the matrix represented a ratio between the area of intersection of one tile of space A with object B over the area of object B. On a yet more detailed level, deep direction relation are represented, where in addition to the empty and non-empty values, the matrix captures when the target object B runs through the neighbouring edge of a tile. This was recorded in a 9 bits representation. The values denoted by each bit location revealed a detailed picture of directional relations for objects of different shapes and extensions.

The breadth of expressiveness, by considering different types of relationships was the target of other works. In [Isl01] the calculus of cardinal direction, proposed by Frank [Fra92], and that of relative orientation, proposed by Freksa [Fre92] were combined. Also qualitative distance was used with directional relations in [HEF95]. It is important to note here that both qualitative and quantitative measures are vital for the development of a practical and effective approach to spatial representation.

2.4 The Spatio-Temporal Domain

The increasing interest and the accumulation of research work in the qualitative spatial domain, together with the well established research in the temporal domain triggered an interest in integrating both domains to investigate the spatio-temporal domain, where objects and their relations change over time.

One of the main approaches here is the extension of the region-based calculus to the spatio-temporal domain. In [Tve93] the RCC-8 theory was extended by considering two dimensional space-time objects presented as rectangles with the RCC-8 eight relations interpreted in space-time. A calculus for representing the location of objects which may move within a certain area has been proposed in [CCB00]. Both the above approaches did not define a set of spatio-temporal relations. In Muller [Mul98], a set of 6 motion classes was given, namely, leave, hit, reach, external, internal and cross. His work is an extension of the RCC-8 theory. In [HC01] 8 spatio-temporal relations were depicted as, immobile, cyclicity, non-cyclicity, coalescence, separation, collision, disjointness and attachment. However, none of the above works presented a systematic exhaustive study for defining JEPD relations (jointly exhaustive and pairwise disjoint) in the spatio-temporal space.

The main attempt to formally define JEPD spatio-temporal relations was carried out by Claramunt [CJ00a, CJ01] using the intersection-based method and Allen temporal relations. A set of 104 relations (71 base with their converse) were defined in 2D space-time and a conceptual neighbourhood analysis was also presented. Figure 2.7 depicts pictorially the basic set of spatio-temporal relations defined in this work.

The above approach was further extended [CJ01] to the 3D spatio-temporal space by considering the temporal relation between intervals as a 4 element matrix representing the start and end of each interval with values of $<$, $=$ or $>$, combining 8 region relations with 7 base relations between intervals (and their converse) as R(region, interval). In the same work the transition between those relations was also investigated. To capture different levels of abstraction in space and time, Claramunt and Jiang [CJ00b] proposed a hierarchical reasoning approach in space and time. Events and spatial regions relations are expressed in a nested fashion with relations on multiple levels.

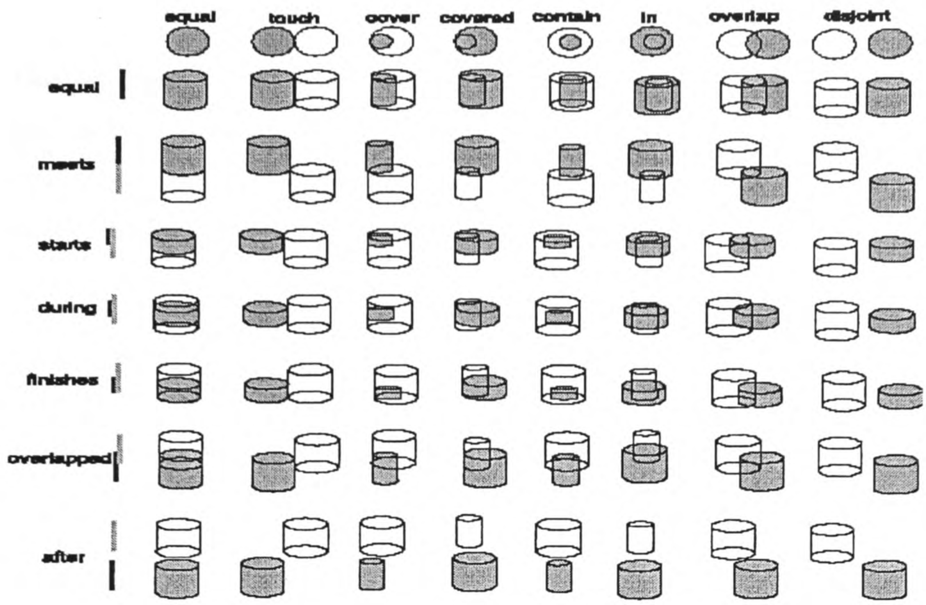


Figure 2.7: Spatio-temporal relations as defined in [CJ00a].

2.5 Spatial Reasoning

As illustrated in figure 1.1, qualitative spatial reasoning can be classified as static or compositional reasoning and dynamic or transitional reasoning. Static reasoning in the topological domain results in the composition tables for specific types of objects. Two main methods were identified in the literature for producing such tables: 1) logical method which is based on axiomatic theories, model generation and theorem proving [CH01], and 2) Algebraic method which is based on set theory and intersection relations [EH90].

Due to the difficulty in producing such tables [RCC92b] and due to the difficulty in identifying the JEPD relations for non-simple objects, composition tables now exists only for region-region relations [EF91], a combinatorial of point, line, regions relations [IMBM00] and occlusion relations [RW02]. An attempt to produce the spatial composition of regions in the the temporal space was presented in [ME02] where the established composition table for regions was used together with the region relations transition graph to compose the relations between A and B in time t_0 and B and C in time t_1, where $t_0 < t_1$.

Composition on direction relations with objects represented as points in conical space division was carried out by Frank [Fra96] while a more detailed composition for direction relations is due to Goyal [Goy00]. However, some discrepancies in the latter method were reported and corrected in [SK01]. In [GR02], two types of spatial relations were combined to give more precise compositions where a combination of topological and size relations was considered.

Dynamic reasoning enjoyed more attention. Transition graphs were produced for simple regions, region-line, and regions with indeterminate boundaries. Also, formal methods for producing such graphs was developed by Galton [Gal94] where a dominance diagram was introduced as a base for transition graph. Also, Egenhofer presented the topological distance between two relations as the difference of the sum of the number of non-empty intersection. At minimum distance, it is hypothesized that the relations are conceptual neighbours, a term coined by Freksa [Fre91a] to express the relation between two relations with continuous change of relations.

2.6 Summary

Following the above review and problem definition, the following observations may be drawn:

1. Topological representation has been limited to simple objects with limited complexity. This limitation may be attributed to limitation in the definition of base ontological primitives to allow for the representation of complexity.

2. Work in the area of QSRR generally follow one of two approaches, a region-based approach and a point-based approach.

3. Automated reasoning based on the region-connection calculus approaches has no

formal basis when the relations are not transitive. This applies to many topological relations and orientation relations.

4. The composition of relations and composition tables were presented only for simple objects (lines in 1D space and areas in 2D space). No attempt was made to construct composition tables for different complex object types. The application of spatial reasoning is not limited to simple cases and consideration of hybrid reasoning between different types of objects which can be complex is needed. One of the main reasons for this shortcoming is the lack of the complete and sound set of relations between objects of different complex types.

5. It is more feasible to arrive at soundness from a completeness-driven formalism by adding general constraints that reflect cognitive aspects of spatial reasoning. Extending the soundness-based approaches towards completeness could not be guaranteed since it depends on human intuition.

6. Point-based approaches have a direct mapping to implementation on the geometric level due to the nature of their atomic relations (intersection) which can be realised directly as geometric operations.

7. In representing varying resolution and different precision, point-based approaches are more general and flexible as they deal directly with the components of the objects. This is particularly important in domains dealing with multiple representation such as geographic domains.

8. Automated reasoning based on connection-based approaches is more difficult since no transitive property is utilised. This is true for various topological and other relations between non simple objects. In the logical based formalisms, such as Randell et al. [RCC92b], the construction of the composition table becomes a real challenge in non-simple cases.

9. In the projection-based methods the relations are considered between the minimum

containing rectangles and not between the objects themselves. This limits their discriminative power in the topological domain and in the cardinal directional domain for interfering rectangles. On the other hand, since the intrinsic orientation relations (left, right, etc) have no mapping to the projection axis, it is difficult to represent them through projection. In fact, the projection method is not totally qualitative when it depends on an external frame of reference (such as the x and y axis) to define the relations. Also, since the relations defined are not precise because of the use of the MBR (a relation such as `disjoint` can be misinterpreted as `inside` or `overlap`), the reasoning results may therefore be false.

10. Spatio-temporal representation can be performed by combining interval and simple region relations. A general representation approach which deals with space and time on equal footings to allow for homogeneous spatio-temporal representation and reasoning to be carried out can prove to be more useful.

11. It is desirable to arrive at a general representation methodology that can be applied to different aspects of qualitative spatial representation of topology, direction and orientation, proximity and uncertainty.

12. The challenge of expressiveness-efficiency trade-off is yet to be addressed formally.

13. Expressiveness of an approach is measured in breadth by its spread over different domains of physical modeling space, while its depth is measured by the coverage of its ontological choices.

14. The work on uncertainty has addressed simple regions only and no work has yet been reported on reasoning with uncertainty.

15. Building composition tables is one of the major challenges for QSRR. Work is needed in the areas of automatic derivation of such tables and in the possibility of conducting spatial reasoning without composition tables in case of incomplete knowledge.

The above summary of findings highlights gaps and challenges currently facing this research area. The goals of this thesis can therefore be rewritten as follows.

1. Develop a qualitative representation method that is capable of representing objects and spatial relationships between objects of arbitrary complexity.

2. Define a general reasoning method based on the representation approach above with no constraints on the type, dimension, or complexity of objects.

3. Extend the representation and reasoning approaches to handle different types of relationships, namely, orientation and proximity relationships.

4. Investigate the applicability of QSRR in the spatio-temporal domain.

5. Investigate the applicability of QSRR in uncertain qualitative spaces.

6. Study the problem of achieving soundness of representation.

Chapter 3

Approaching Complexity in Spatial Representation and Efficiency in Spatial Reasoning

The first part of this chapter addresses the problem of qualitative representation of objects with arbitrary spatial complexity and the representation of topological relationships. Different types of object are handled, and the extension of the approach to composite objects formed of distinct parts is proposed. In the second part of the chapter, a reasoning formalism is proposed, which is based on the representation method above, for the automatic derivation of spatial relationships. The reasoning approach consists of two parts: a) general constraints governing the spatial relationships between objects in space, and b) general rules to propagate relationships between those objects. Both the constraints and the rules are based on a uniform representation of the topology of the objects, their embedding space and the representation of the relationships between them.

3.1 The General Representation Formalism

Objects of interest and their embedding space are divided into components according to a required resolution. The connectivity of those components is explicitly represented.

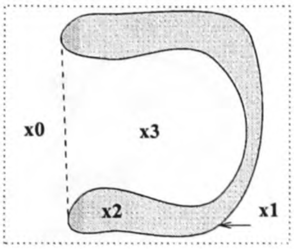
Spatial relations are represented by the intersection of object components in a similar fashion to that described in [EH90] but with no restriction on object components to consist only of two parts (boundary and interior).

3.1.1 The Underlying Representation of Object Topology

Let S be the space in which the object is embedded. The object and its embedding space are assumed to be *dense* and *connected*. The embedding space is also assumed to be infinite. The object and its embedding space are decomposed into components partitioning the space and reflecting the objects and space topology such that,

1. No overlap exists between any of the representative components.
2. The union of the components is equal to the embedding space.

The topology of the object and the embedding space can then be described by a matrix whose elements represent the connectivity relations between its components. This matrix shall be denoted *adjacency matrix*. In figure 3.1(a) a possible decomposition of a concave shaped object (for example an island with a bay) and its embedding space is shown and in 3.1(b) the adjacency matrix for its components is presented. The object is represented by two components a linear component x_1 (the shore line of the island) and an areal component x_2 and the rest of its embedding space is represented by a finite areal component x_3 (representing the bay of the island) and infinite areal component x_0 representing the surrounding area. The fact that two components are connected is represented by a (1) in the adjacency matrix and by a (0) otherwise. Since connectivity is a symmetric relation, the resulting matrix will be symmetric around the diagonal. Hence, only half the matrix is sufficient for the representation of the object's topology and the matrix can be collapsed to the structure in figure 3.1(c). In the decomposition strategy, the complement of the object in question shall be considered to be infinite. The suffix 0 (x_0) is used to represent this component.



(a)

	x_0	x_1	x_2	x_3
x_0	-	1	0	1
x_1	1	-	1	1
x_2	0	1	-	0
x_3	1	1	0	-

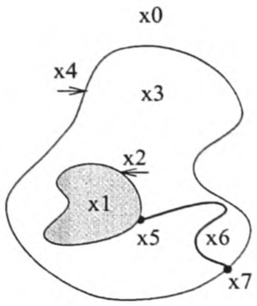
(b)

x_0			
1	x_1		
0	1	x_2	
1	1	0	x_3

(c)

Figure 3.1: (a) Possible decomposition of a concave-shaped object and its embedding space. (b) Adjacency matrix of the shape in (a). (c) Half the symmetric adjacency matrix is sufficient to capture the object representation.

Note that different decomposition strategies for the objects and their embedding spaces can be used according to the precision of the relations required and the specific application considered. The higher the resolution used (or the finer the components of the space and the objects), the higher the precision of the resulting set of relations in the domain considered. For example, consider the objects in figure 3.2(a) which represents an island with a lake represented by x_1 and x_2 and a river represented by the components x_5, x_6 and x_7. The adjacency matrix for the map in (a) in given in (b). This example demonstrates the ability of the adjacency structure to represent complex objects such as a whole map. At a lower resolution the river object may be omitted by removing the rows and columns of components x_5, x_6 and x_7. This representation can also be used to represent virtual components as was seen in figure 3.1 which makes the method flexible for representation in any application domain. In geographic databases, objects are usually associated by scales of representation, and can possess different spatial representations under different scales. For example, a regiòn can collapse to a single point in small scale maps. In these cases, the objects' components will need to be changed to reflect this effect.



(a)

x_0							
0	x_1						
0	1	x_2					
0	0	1	x_3				
1	0	0	1	x_4			
0	1	1	1	0	x_5		
0	0	0	1	0	1	x_6	
1	0	0	1	1	0	1	x_7

(b)

Figure 3.2: Representation of a complex object of a map consisting of an island with a lake x_1 and x_2 and a river x_5, x_6 and x_7 by the adjacency matrix.

3.1.2 The Underlying Representation of Spatial Relations

In this section, the representation of the topological relations through the intersection of their components [AA95] is adopted and generalized for objects of arbitrary complexity.

Distinction of topological relations is dependent on the strategy used in the decomposition of the objects and their related spaces. For example, in figure 3.3 different relationships between two objects representing a ship (x) and an island (y) are shown, where in 3.3(a) the ship is outside the bay and in 3.3(b) the ship is inside the bay. The concave region representing the island (y) is decomposed into two components y_1 and y_2 and the rest of the space associated with y is decomposed into two components (y_3 representing the bay and y_0 representing the rest of the ocean). Note that the component y_3 is a virtual component, i.e. with no physical boundary to delineate its spatial extension. It is the identification of this component that makes the distinction between the two relationships in the figure. The complete set of spatial relationships are represented by combinatorial intersection of the components of one space with those of the other space.

If $R(x, y)$ is a relation of interest between object x and object y, and X and Y are the

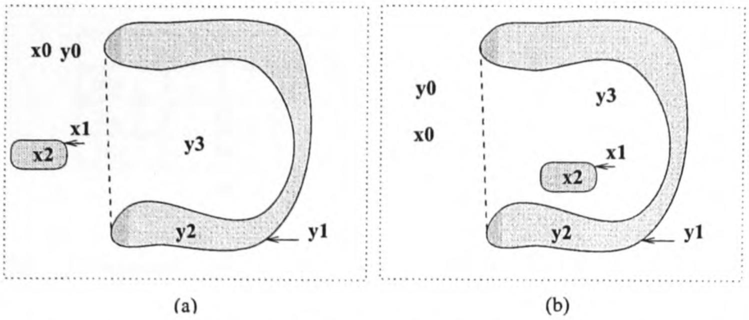


Figure 3.3: Different qualitative spatial relationships can be distinguished by identifying the appropriate components of the objects and the space.

spaces associated with the objects respectively such that m is the number of components in X and l is the number of components in Y, then a spatial relation $R(x, y)$ can be represented by one state of the following equation:

$$\begin{aligned} R(x,y) &= X \cap Y \\ &= \left(\bigcup_{i=1}^{m} x_i\right) \cap \left(\bigcup_{j=1}^{l} y_j\right) \\ &= (x_1 \cap y_1, \cdots, x_1 \cap y_l, x_2 \cap y_1, \cdots, x_m \cap y_l) \end{aligned}$$

The intersection $x_i \cap y_j$ can be an empty or a non-empty intersection. The above set of intersections shall be represented by an intersection matrix, as follows.

$R(x, y) =$

	y_0	y_1	y_2	$\cdots$
x_0				
x_1				
x_2				
$\vdots$				

For example, the intersection matrices corresponding to the spatial relationships in figure 3.3 are shown in figure 3.4. The components x_1 and x_2 have a non-empty intersection with y_0 in 3.4(a) and with y_3 in 3.4(b).

	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	1	0	0	0
x_2	1	0	0	0

(a)

	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	0	0	0	1
x_2	0	0	0	1

(b)

Figure 3.4: The corresponding intersection matrices for the relationships in figure 3.3 respectively.

Different combinations in the intersection matrix can represent different qualitative relations. The set of valid or sound spatial relationships between objects is dependent on the particular domain studied. For example, in considering relationships between two line objects in a network analysis application we might be interested in only those relationships where end points of lines are in contact. Also, properties of the objects would affect the set of possible spatial relationships that can exist between them. For example, if one object is solid and the other is permeable, there cannot be any intersection of the inside of the solid object with any other component of the other object. Also, objects of different size or shape cannot be involved in certain spatial relations such as `equal` or `contain` between the smaller and the larger object.

The example in figure 3.5 demonstrates the six possible spatial relations that can exist between two *solid* objects, one having the shape of a convex region and the other a concave one along with their intersection matrices. Note that since y_3 is a virtual area component, no boundary is defined for it, i.e. no boundary is defined between y_3 and y_0. Thus a relation where $x_1 \cap y_3 = 1 \ \wedge \ x_2 \cap y_3 = \phi$ is not considered. If y_3 has a non-empty intersection with x_1 (a linear component) then it has also to intersect with x_2 producing the relation R_3 in the figure. The example can be used to represent many situations, for example, a solid object falling into a container full of liquid, a ball thrown into a net, or a ship entering a bay of an island, etc. Note that since object y is a solid object, the component y_2 will always have only one intersection relation with x_0.

$R_1 =$

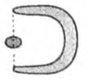
	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	1	0	0	0
x_2	1	0	0	0

$R_2 =$

	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	1	1	0	0
x_2	1	0	0	0

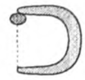
$R_3 =$

	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	1	0	0	1
x_2	1	0	0	1



$R_4 =$

	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	1	1	0	1
x_2	1	0	0	1



$R_5 =$

	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	0	0	0	1
x_2	0	0	0	1

$R_6 =$

	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	0	1	0	1
x_2	0	0	0	1

Figure 3.5: A set of 6 spatial relationships between two solid bodies. The decomposition of objects is as in figure 3.3.

3.2 The General Reasoning Formalism

The reasoning approach consists of: a) general constraints to govern the spatial relationships between objects in space, and b) general rules to propagate relationships between the objects.

3.2.1 General Constraints

The intersection matrix is in fact a set of constraints whose values identifies specific spatial relationships. For example, part of the constraints used to represent the relationship in figure 3.3(a) are $x_1 \cap y_1 = 0, x_1 \cap y_2 = 0, x_1 \cap y_3 = 0, x_2 \cap y_0 = 1, \cdots$

The process of spatial reasoning can be defined as the process of propagating the constraints of two spatial relations (for example, $R_1(A, B)$ and $R_2(B, C)$), to derive a new set of constraints between objects A and C. The derived constraints can then be mapped to a specific spatial relation (i.e. the relation $R_3(A, C)$).

A subset of the set of constraints defining all spatial relations are general and are applicable to any relationship between any objects. These general constraints are a consequence of the initial assumptions used in the definition of the object and space topology. The identification of these constraints complements the reasoning rules and shall be used later in the thesis to give some insight in the propagation of spatial relations.

The two general constraints are:

1. Every unbounded (infinite) component of one space must intersect with at least one unbounded (infinite) component of the other space.

   Intuitively this rule says that it is impossible for an infinite component in the space to only have an intersection with finite component(s). In this case the infinite component becomes a subset of the finite component(s) which is not possible. In figure 3.5, x_0 and y_0 always have a non-empty intersection.

2. Every component from one space must intersect with at least one component from the other space.

   If one component of one space does not intersect with any component of the other space, either the two spaces are not equal or the spaces are not dense or *connected.* Both conditions are excluded by the initial assumptions. This implies that there cannot exist a row or a column in the intersection matrix whose elements are all empty intersections, hence the combinatorial cases in the matrix where this case exists can be ignored.

Domain Specific Constraints

The above general constraints are applicable for any object type in any application domain. Domain specific constraints can be used to determine the set of relations which are physically possible between the objects under consideration. For example, in a CAD system where it is required to determine the set of possible assemblies between two objects, one of which containing a cylindrical hole S_1 with diameter d_1 and the other comprising a solid cylinder S_2 with diameter d_2 and $d_2 > d_1$. In this case a domain specific constraint will be that $S_2 \cap S_1 = \phi$. Object characteristics such as shape, dimension, permeability, etc. can be used to define specific constraints.

In this case the formalism can be used in one of two modes.

1. Either identify all the domain specific constraints and use them to prune the set of combinatorial intersections into the physically feasible relations, for example, such rules were used to derive spatial relations between a region and a line in [JB94]. This step is essential if complete composition tables are to be derived.

2. Or use the domain specific constraints to determine the set of possible relations every time a spatial composition is performed.

3.2.2 General Reasoning Rules

Composition of spatial relations is the process through which the possible relationship(s) between two objects x and z is derived given two relationships: R_1 between x and y and R_2 between y and z. Two general reasoning rules for the propagation of intersection constraints are presented. The rules are characterized by the ability to reason over spatial relationships between objects of arbitrary complexity in any space dimension. These rules allow for the automatic derivation of the composition (transitivity) tables between any spatial shapes - a task considered to be a challenge to automatic theorem provers [RCC92b].

Reasoning Rules

Composition of spatial relations using the *intersection* representation approach is based on the transitive property of the subset relations. In what follows the following subset notation is used. If x' is a set of components (non empty sets of point-sets) $\{x_1, \cdots, x_{m'}\}$ in a space X, and y_j is a component in space Y, then $\sqsubseteq$ denotes the following subset relationship.

- $y_j \sqsubseteq x'$ denotes the subset relationship such that: $\forall x_i \in x'(y_j \cap x_i \neq \phi) \wedge y_j \cap (X - x_1 - x_2 \cdots - x_{m'}) = \phi \ where \ i = 1, \cdots m'$. Intuitively, this symbol indicates that the component y_j intersects with every set in the collection x' and does not intersect with any set outside of x'. That is, x' is the minimal cover of y_j from the partition X.

If x_i, y_j and z_k are components of spaces X, Y and Z respectively, then if there is a non-empty intersection between x_i and y_j, and y_j is a subset of z_k, then it can be concluded that there is also a non-empty intersection between x_i and z_k.

$$(x_i \cap y_j \neq \phi) \ \wedge \ (y_j \subseteq z_k) \rightarrow (x_i \cap z_k \neq \phi)$$

This relation and the transitivity of the subset relations can be generalized in the following two rules. The rules describe the propagation of intersections between the components of objects and their related spaces involved in the spatial composition.

Rule 1: Propagation of Non-Empty Intersections

Let $x' = \{x_1, x_2, \cdots, x_{m'}\}$ be a subset of the set of components of space X whose total number of components is m and $m' \leq m$; $x' \subseteq X$. Let $z' = \{z_1, z_2, \cdots, z_{n'}\}$ be a subset of the set of components of space Z whose total number of components is n and $n' \leq n$; $z' \subseteq Z$. If y_j is a component of space Y, the following is a governing rule of interaction for the three spaces X, Y and Z.

$$\begin{aligned}
(x' \sqsupseteq y_j) \quad & \wedge \quad (y_j \sqsubseteq z') \\
& \rightarrow \quad (x' \cap z' \neq \phi) \\
& \equiv \quad (x_1 \cap z_1 \neq \phi \vee \cdots \vee x_1 \cap z_{n'} \neq \phi) \wedge (x_2 \cap z_1 \neq \phi \vee \cdots \vee x_2 \cap z_{n'} \neq \phi) \\
& \qquad \wedge \cdots \wedge (x_{m'} \cap z_1 \neq \phi \vee \cdots \vee x_{m'} \cap z_{n'} \neq \phi) \\
& \qquad \wedge (z_1 \cap x_1 \neq \phi \vee \cdots \vee z_1 \cap x_{m'} \neq \phi) \wedge (z_2 \cap x_1 \neq \phi \vee \cdots \vee z_2 \cap x_{m'} \neq \phi) \\
& \qquad \wedge \cdots \wedge (z_{n'} \cap x_1 \neq \phi \vee \cdots \vee z_{n'} \cap x_{m'} \neq \phi)
\end{aligned}$$

The above rule states that if the component y_j in space Y has a non-empty intersection with every component from the sets x' and z', then each component of the set x' must intersect with at least one component of the set z' and vice versa.

It follows that if either $m' = 1$ or $n' = 1$ or both, then the result of the propagation will be definite, i.e. $x_1 \sqsupseteq y_j \wedge y_j \sqsubseteq z' \rightarrow (x_1 \cap z_1 \neq \phi \wedge x_1 \cap z_2 \neq \phi \cdots \wedge x_1 \cap z_{n'} \neq \phi)$. Similarly, if $x' \sqsupseteq y_j \wedge y_j \sqsubseteq z_1 \rightarrow (z_1 \cap x_1 \neq \phi \wedge z_1 \cap x_2 \neq \phi \cdots \wedge z_1 \cap x_{m'} \neq \phi)$.

The proof is as follows: using the definition of $\sqsubseteq$ it follows that $y_j \sqsubseteq x_1 \rightarrow \exists x'_1(x'_1 \subseteq x_1 \wedge x'_1 = y_j)$ and that $y_j \sqsubseteq z' \rightarrow \exists(z'_1, z'_2, \cdots, z'_n)(z'_1 \subseteq z_1 \wedge z'_2 \subseteq z_1 \cdots \wedge z'_n \subseteq z_1 \wedge y_j = (z'_1 \cup z'_2 \cdots \cup z'_n))$ since y_j intersects with every component in z'. From the above it can be concluded that $x'_1 = z'_1 \cup z'_2 \cdots \cup z'_n$, i.e. x'_1 intersects with each component in the set z'.

In the case where $m' \neq 1 \wedge n' \neq 1$, then the constraint $x_i \cap z_1 \neq \phi \vee x_i \cap z_2 \neq \phi \cdots \vee x_i \cap z_{n'} \neq \phi$ can be expressed in the intersection matrix by a label, for example the label a_r $(r = 1, 2, \cdots)$ in the following matrix indicates $x_1 \cap (z_2 \cup z_4) \neq \phi$ (x_1 has a positive intersection with z_2, or with z_4 or with both). A ? in the matrix indicates that the intersection is either empty or non-empty.

	z_1	z_2	z_3	z_4	$\cdots$	z_n
x_1	?	a_1	?	a_2	?	?

Rule 1 represents the propagation of non-empty intersections of components in space. A rule for the propagation of empty intersections can be stated as follows.

Rule 2: Propagation of Empty Intersections

Let $z' = \{z_1, z_2, \cdots, z_{n'}\}$ be a subset of the set of components of space Z whose total number of components is n and $n' < n$; $z' \subseteq Z$. Let $y' = \{y_1, y_2, \cdots, y_{l'}\}$ be a subset of the set of components of space Y whose total number of components is l and $l' < l$; $y' \subseteq Y$. Let x_i be a component of the space X. Then the following is a governing rule for the spaces X, Y and Z. Note that $X = Y = Z$ are representations of the Universal set for space.

$$(x_i \sqsubseteq y') \quad \wedge \quad (y' \sqsubseteq z')$$

$$\begin{aligned} &\rightarrow \quad x_i \subseteq z' \\ &\rightarrow \quad x_i \not\subseteq (Z - z') \\ &\equiv \quad (x_i \cap (Z - z_1 - z_2 \cdots - z_{n'}) = \phi) \end{aligned}$$

Similarly,

$$\begin{aligned} (z_k \sqsubseteq y') \quad \wedge \quad & (y' \sqsubseteq x') \\ \rightarrow \quad & z_k \subseteq x' \\ \rightarrow \quad & z_k \not\subseteq (X - x') \\ \equiv \quad & (z_k \cap (X - x_1 - x_2 \cdots - x_{m'}) = \phi) \end{aligned}$$

Remark: if $n' = n$, i.e. x_i may intersect with every element in Z, or if $m' = m$, i.e. z_k may intersect with every element in X, or if $l' = l$, i.e. x_i (or z_k) may intersect with every element in Y, then no empty intersections can be propagated. Rules 1 and 2 are the two general rules for propagating empty and non-empty intersections of components of spaces.

Note that in both rules the intermediate object (y) and its space components plays the main role in the propagation of intersections. Indeed, it shall be shown in the examples how the rule 1 is applied a number of times equal to the number of components of the space of the intermediate object. Hence, the composition of spatial relations using this method becomes a tractable problem which can be performed in a defined limited number of steps.

Soundness and Completeness of the Formalism

The formalism can be said to be sound if any derived conclusion using the rules follows set-theoretically, and the formalism can be said to be complete if any conclusions which follows semantically from the axioms of the set theory are also derivable by the formalism.

In this section the formalism is proved to be sound and complete using the basic axioms of transitivity and set intersections in the set theory, in particular,

- transitivity of subsets: $A \subseteq B \subseteq C \rightarrow A \subseteq C$, and its implication: $A \subseteq C \rightarrow$

$A \cap (C^*) = \phi$, where C^* is the complement of C.

- set intersection: $A \cap B \ \wedge \ B \subseteq C \rightarrow A \cap C \neq \phi$, and, $C \cap B \ \wedge \ B \subseteq A \rightarrow A \cap C \neq \phi$. These rules can be derived directly from the transitivity axiom as follows: If $\exists \alpha(\alpha \in A \wedge \alpha \in B)$ then $(\alpha \subseteq A) \wedge (\alpha \subseteq B) \wedge (B \subseteq C) \rightarrow \alpha \subseteq C$ or $\alpha \cap C \neq \phi$. Hence, $A \cap B \wedge B \subseteq C \rightarrow A \cap C \neq \phi$.

Soundness of the formalism: Rule 1 states that

$$\begin{aligned}
(x' \sqsupseteq y_j) \quad \wedge & \quad (y_j \sqsubseteq z') \\
\rightarrow & \quad (x' \cap z' \neq \phi) \\
\equiv & \quad (x_1 \cap z_1 \neq \phi \vee \cdots \vee x_1 \cap z_{n'} \neq \phi) \qquad i.e.(x_1 \cap z' \neq \phi) \\
& \quad \wedge \cdots \\
& \quad \wedge(x_{m'} \cap z_1 \neq \phi \vee \cdots \vee x_{m'} \cap z_{n'} \neq \phi) \qquad i.e.(x_{m'} \cap z' \neq \phi) \\
& \quad (z_1 \cap x_1 \neq \phi \vee \cdots \vee z_1 \cap x_{m'} \neq \phi) \quad i.e.(z_1 \cap x' \neq \phi) \\
& \quad \wedge \cdots \\
& \quad \wedge(z_{n'} \cap x_1 \neq \phi \vee \cdots \vee z_{n'} \cap x_{m'} \neq \phi) \quad i.e.(z_{n'} \cap x' \neq \phi)
\end{aligned}$$

Since $x' \sqsupseteq y_j \rightarrow (y_j \cap x_1 \neq \phi \wedge y_j \cap x_2 \neq \phi \wedge \cdots \wedge y_j \cap x_{m'} \neq \phi)$, and, $z' \sqsupseteq y_j \rightarrow (y_j \cap z_1 \neq \phi \wedge y_j \cap z_2 \neq \phi \wedge \cdots \wedge y_j \cap z_{n'} \neq \phi)$, then, rule 1 can be expressed by the collection of the following axioms:

$$\begin{aligned}
x_1 \cap y_j \neq \phi \wedge y_j \sqsubseteq z' & \rightarrow x_1 \cap z' \neq \phi \\
x_2 \cap y_j \neq \phi \wedge y_j \sqsubseteq z' & \rightarrow x_2 \cap z' \neq \phi \\
& \vdots \\
x_{m'} \cap y_j \neq \phi \wedge y_j \sqsubseteq z' & \rightarrow x_{m'} \cap z' \neq \phi \\
z_1 \cap y_j \neq \phi \wedge y_j \sqsubseteq x' & \rightarrow z_1 \cap x' \neq \phi \\
z_2 \cap y_j \neq \phi \wedge y_j \sqsubseteq x' & \rightarrow z_2 \cap x' \neq \phi \\
& \vdots \\
z_{n'} \cap y_j \neq \phi \wedge y_j \sqsubseteq x' & \rightarrow z_{n'} \cap x' \neq \phi
\end{aligned}$$

Hence, rule 1 reduces to the axiom of set intersection and is therefore sound.

Rule 2 states that:

$$(x_i \sqsubseteq y') \quad \wedge \quad (y' \sqsubseteq z') \rightarrow (x_i \cap (Z - z') = \phi)$$

$Z - z'$ is the complement of z'. Using the transitivity of subsets, $x_i \sqsubseteq y' \wedge y' \sqsubseteq z' \rightarrow x_i \subseteq z'$, then intersection of x_i with the complement of z' must be empty. Hence rule 2 is also sound.

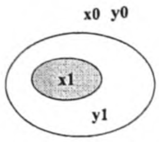
Completeness of the formalism: As shown above, rule 1 is an equivalent form of the set intersection axiom and hence any conclusion which can be derived using this axiom is also derivable using rule 1.

From the set theory we have that: $A \subseteq B \subseteq C \rightarrow A \subseteq C \rightarrow A \cap (U - C) = \phi$, where U is the universal set for space. In the formalism the underlying spaces for the objects are equal, i.e. $X = Y = Z$ and all are equivalent to the Universal set for space. Hence, $\forall x \in X(x \subseteq Z)$, and similarly, $\forall z \in Z(z \subseteq X)$. From rule 2 we have that: $x_i \sqsubseteq y' \sqsubseteq z' \rightarrow x_i \subseteq z' \rightarrow x_i \cap (Z - z') = \phi$ where Z is the universal set for space. Then rule 2 reduces to the subset transitivity axiom and its implication, and any conclusion which can be derived using these axioms are also derivable by the formalism.

Since both rules in the formalism are equivalent to basic axioms of the set theory, then the formalism is set-theoretically complete with respect to the two axioms and any axioms derived from them.

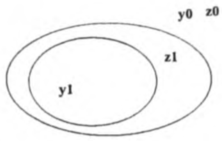
3.2.3 Analysis of the Formalism

If m' and n' are the number of components of the sets x' and z' respectively and m and n are the total number of components of the spaces X and Z respectively and $x' \sqsubseteq X$ and $z' \sqsubseteq Z$. Using rule 1 the composition of relations can be classified into the following.



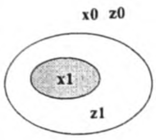
	y_0	y_1
x_0	1	1
x_1	0	1

(a)



	z_0	z_1
y_0	1	1
y_1	0	1

(b)



	z_0	z_1
x_0	1	1
x_1	0	1

(c)

Figure 3.6: (a) *inside*(x, y) and the corresponding intersection matrix. (b) *inside*(y, z) and the corresponding intersection matrix. (c) The resulting *inside* relationship between x and z.

I. If $(m' = 1 \lor n' = 1)$, (e.g. $x' = \{x_1\}$ or $z' = \{z_1\}$ or both) then the rule shall propagate a definite set of intersections. For example, if y_j intersects the only element of x', then this element of x' must have a non-empty intersection with every element from the set z'. Also, if y_j intersects with the only element of z', then this element of z' must have a non-empty intersection with every element from the set x'. If this property holds for every component of the intermediate space Y then the composition must result in a definite relation. An example of this case is the composition of the inside relationship between two simple convex regions as shown below.

Example

Consider the simple example of composing the inside relationships between convex regions x, y and z. The regions and their corresponding intersection matrices are shown in figure 3.6. Note that the regions are defined as wholes with no distinction of boundaries and interiors.

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- y_0 intersections: $(m' = 1$ and $n' = 2)$

$$\{x_0\} \sqsupseteq y_0 \wedge y_0 \sqsubseteq \{z_0, z_1\} \quad \rightarrow \quad z_0 \cap x_0 \neq \phi \wedge z_1 \cap x_0 \neq \phi$$

- y_1 intersections: $(m' = 2$ and $n' = 1)$

$$\{x_0, x_1\} \sqsupseteq y_1 \wedge y_1 \sqsubseteq \{z_1\} \quad \rightarrow \quad x_0 \cap z_1 \neq \phi \wedge x_1 \cap z_1 \neq \phi$$

Applying rule 2 we get the following,

- $x_1 \sqsubseteq \{y_1\} \sqsubseteq \{z_1\} \rightarrow x_1 \cap z_0 = \phi$

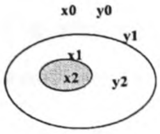
Grouping the above constraints, we get the intersection matrix and relationship in figure 3.6(c).

II. If $(m' > 1 \ \wedge \ n' > 1)$, (e.g. if $x' = \{x_1, x_2\}$ and $z' = \{z_1, z_2\}$), for at least one y_j of the space Y, no definite intersections are propagated (i.e. $x' \cap z' \neq \phi$). If after the application of the reasoning rules this result still holds, then the composition shall produce a non-definite set of disjunctive relations.

III. If $(m' = m \ \wedge \ n' = n)$, i.e. $(X \sqsupseteq y_j) \ \wedge \ (y_j \sqsubseteq Z)$, no distinguishing constraints can be propagated from the component y_j, as this case is an expression of the second general constraint. Also since the implication of such constraint is that every component of one space may intersect with all the components of the other space no empty intersection will be propagated (using rule 2) for any component.

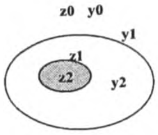
IV. If $(m' = 1 \ \wedge \ n' = 1 \ \wedge \ x' = \{x_0\} \ \wedge \ z' = \{z_0\})$, i.e. x' is the infinite component and z' is the infinite component, then the rule becomes an expression of the first general constraint, i.e. no distinguishing constraint will be propagated.

V. If all the propagated intersections for the set of components of the intermediate space are either of type 3 or 4 above or both then the composition results in the universal relation (disjunction of all possible relationships) - since the only constraints propagated are the general ones, i.e no specific constraint is propagated. An example is given below.



	y_0	y_1	y_2
x_0	1	1	1
x_1	0	0	1
x_2	0	0	1

(a)



	z_0	z_1	z_2
y_0	1	0	0
y_1	1	0	0
y_2	1	1	1

(b)

Figure 3.7: (a) The relationship *inside*(x, y) and the corresponding intersection matrix. (b) The relationship *contain*(y, z) and the corresponding intersection matrix.

Example: Demonstration of the General Constraints

Consider the relationships between convex regions x, y and z to be as shown in figure 3.7.

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- y_0 intersections: $(m' = 1$ and $n' = 1)$

$$\{x_0\} \sqsupseteq y_0 \wedge y_0 \sqsubseteq \{z_0\} \quad \rightarrow \quad x_0 \cap z_0 \neq \phi$$

- y_1 intersections: $(m' = 1$ and $n' = 1)$

$$\{x_0\} \sqsupseteq y_1 \wedge y_1 \sqsubseteq \{z_0\} \quad \rightarrow \quad x_0 \cap z_0 \neq \phi$$

- y_2 intersections: $(m' = m$ and $n' = n)$

$$\{x_0, x_1, x_2\} \sqsupseteq y_2 \wedge y_2 \sqsubseteq \{z_0, z_1, z_2\} \quad \rightarrow \quad X \cap Z \neq \phi$$

Applying rule 2 we get the following,

- $x_0 \sqsubseteq Y \sqsubseteq Z$ No empty intersections can be propagated.
- $x_1 \sqsubseteq \{y_2\} \sqsubseteq Z$ No empty intersections can be propagated.
- $x_2 \sqsubseteq \{y_2\} \sqsubseteq Z$ No empty intersections can be propagated.

The two general constraints are the only constraints propagated in this example and that is why the result of the composition is a disjunction of all possible relations between the two regions. The result matrix is as follows.

	z_0	z_1	z_2
x_0	1	?	?
x_1	?	?	?
x_1	?	?	?

Similar results can be obtained for $overlap(A, B) \wedge overlap(B, C)$ and $disjoint(A, B) \wedge disjoint(B, C)$ for simple convex polygons.

3.2.4 Example of Spatial Reasoning with Complex Objects

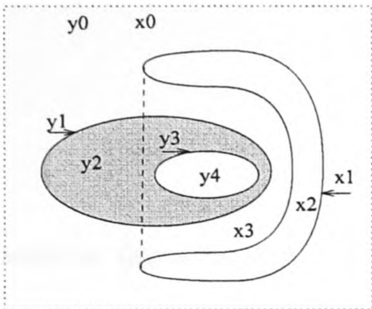
The example in figure 3.8 is used for demonstrating the composition of relations using non-simple spatial objects. Figure 3.8(a) shows the relationship between a concave region x and a region with a hole y and 3.8(b) shows the relationship between object y and a simple convex region z where z touches the the hole in y. The intersection matrices corresponding to the two relationships are also shown.

Given that the possible set of relationships that can occur between x and z in a certain domain are as shown in figure 3.5, it is required to derive the possible relationships between these two objects given the situation in figure 3.8.

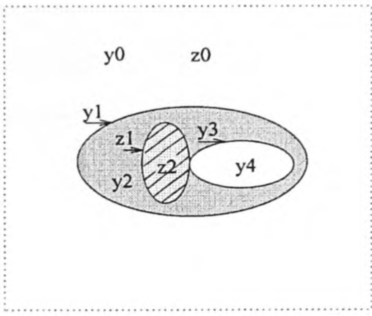
The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- y_0 intersections: ($m' = 4$ and $n' = 1$)

$$
\begin{aligned}
\{x_0, x_1, x_2, x_3\} \sqsupseteq y_0 \quad & \wedge \quad y_0 \sqsubseteq \{z_0\} \\
& \rightarrow \quad x_0 \cap z_0 \neq \phi \wedge x_1 \cap z_0 \neq \phi \\
& \wedge \quad x_2 \cap z_0 \neq \phi \wedge x_3 \cap z_0 \neq \phi
\end{aligned}
$$



(a)



(b)

	y_0	y_1	y_2	y_3	y_4
x_0	1	1	1	0	0
x_1	1	0	0	0	0
x_2	1	0	0	0	0
x_3	1	1	1	1	1

(c)

	z_0	z_1	z_2
y_0	1	0	0
y_1	1	0	0
y_2	1	1	1
y_3	1	1	0
y_4	1	0	0

(d)

Figure 3.8: (a) and (b) Spatial relationships between non-simple objects x, y and z. (c) and (d) Corresponding intersection matrices.

- y_1 intersections: $(m' = 2$ and $n' = 1)$

$$\{x_0, x_3\} \sqsupseteq y_1 \wedge y_1 \sqsubseteq \{z_0\} \quad \rightarrow \quad x_0 \cap z_0 \neq \phi \wedge x_3 \cap z_0 \neq \phi$$

- y_2 intersections: $(m' = 2$ and $n' = 3)$

$$\begin{aligned} \{x_0, x_3\} \sqsupseteq y_2 \quad & \wedge \quad y_2 \sqsubseteq \{z_0, z_1, z_2\} \\ & \rightarrow \quad x_0 \cap Z \neq \phi \ \wedge \ x_3 \cap Z \neq \phi \\ & \wedge \quad (z_0 \cap x_0 \neq \phi \vee z_0 \cap x_3 \neq \phi) \\ & \wedge \quad (z_1 \cap x_0 \neq \phi \vee z_1 \cap x_3 \neq \phi) \\ & \wedge \quad (z_2 \cap x_0 \neq \phi \vee z_2 \cap x_3 \neq \phi) \end{aligned}$$

- y_3 intersections: $(m' = 1$ and $n' = 2)$

$$\begin{aligned} \{x_3\} \sqsupseteq y_3 \quad & \wedge \quad y_3 \sqsubseteq \{z_0, z_1\} \\ & \rightarrow \quad (z_0 \cap x_3 \neq \phi \wedge z_1 \cap x_3 \neq \phi) \end{aligned}$$

- y_4 intersections: $(m' = 1$ and $n' = 1)$

$$\{x_3\} \sqsupseteq y_4 \ \wedge \ y_4 \sqsubseteq \{z_0\} \quad \rightarrow \quad x_3 \cap z_0 \neq \phi$$

Applying rule 2 we get the following,

- $x_0 \sqsubseteq \{y_0, y_1, y_2\} \ \wedge \ \{y_0, y_1, y_2\} \sqsubseteq \{z_0, z_1, z_2\}$

  No empty intersections can be propagated between x_0 and Z.

- $x_1 \sqsubseteq y_0 \ \wedge \ y_0 \sqsubseteq \{z_0\} \rightarrow x_1 \cap z_1 = \phi \wedge x_1 \cap z_2 = \phi$

- $x_2 \sqsubseteq y_0 \ \wedge \ y_0 \sqsubseteq \{z_0\} \rightarrow x_2 \cap z_1 = \phi \wedge x_2 \cap z_2 = \phi$

- $x_3 \sqsubseteq \{y_0, y_1, y_2, y_3, y_4\} \ \wedge \ \{y_0, y_1, y_2, y_3, y_4\} \sqsubseteq \{z_0, z_1, z_2\}$

  No empty intersections can be propagated between x_3 and Z.

Note that for the intersections of y_2 rule 1 is applied once from the point of view of object x, and then from the point of view of object z to satisfy the rule. The constraints propagated are then refined and the stronger constraints selected. Refining the above constraints, we get the following intersection matrix.

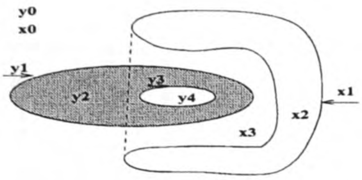
	z_0	z_1	z_2
x_0	1	?	a_1
x_1	1	0	0
x_2	1	0	0
x_3	1	1	a_2

Comparing the resulting matrix above with the matrices in figure 3.5, it can be seen that the result matrix corresponds to two possible relationships between objects x and z, namely the relationships R_3 and R_5. The result of the composition is indefinite which complies with analysis point II, section 2.3, where both m' and n' are greater than 1 ($m' = 2$ and $n' = 3$ for y_2).

A different conclusion is obtained if the relationship between objects y and z is as shown in figure 3.9(b). The composition of the relationships between x, y and z in this case will result in the definite matrix in figure 3.9(c) which corresponds to R_5 in figure 3.5. In this case the intersections of the components for space Y will be as follows: ($y_0 : m' = 4 \wedge n' = 1$, $y_1 : m' = 2 \wedge n' = 1$, $y_2 : m' = 2 \wedge n' = 1$, $y_3 : m' = 1 \wedge n' = 1$, $y_4 : m' = 1 \wedge n' = 3$), i.e. $m' = 1$ or $n' = 1$ or both in all the intersections implying a definite composition result according to point I in the analysis.

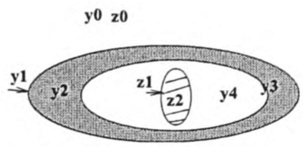
3.2.5 Reasoning between Objects with Different Dimensions

Spatial reasoning is needed between spatial objects of different dimension and not only between objects with similar dimension. The set of valid relations between regions and between lines and regions have been identified [JB94]. As an example of reasoning between regions and lines is shown in figure 3.10. The matrices for the relations in the figure are



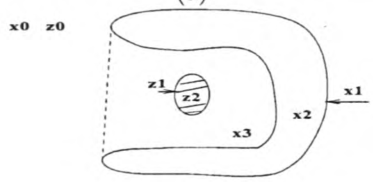
	y_0	y_1	y_2	y_3	y_4
x_0	1	1	1	0	0
x_1	1	0	0	0	0
x_2	1	0	0	0	0
x_3	1	1	1	1	1

(a)



	z_0	z_1	z_2
y_0	1	0	0
y_1	1	0	0
y_2	1	0	0
y_3	1	0	0
y_4	1	1	1

(b)



	z_0	z_1	z_2
x_0	1	0	0
x_1	1	0	0
x_2	1	0	0
x_3	1	1	1

(c)

Figure 3.9: Given the relationship between objects x and y in (a) and the relation between the y and z in (b), the composition results in the definite intersection matrix between x and z shown in (c).

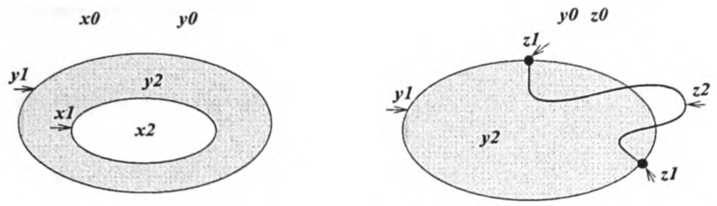


Figure 3.10: Relationships between object with different dimension.

as follows.

$$R(x,z) = \begin{array}{c|c|c|c} & y_0 & y_1 & y_2 \\ \hline x_0 & 1 & 1 & 1 \\ x_1 & 0 & 0 & 1 \\ x_2 & 0 & 0 & 1 \end{array} \circ \begin{array}{c|c|c|c} & z_0 & z_1 & z_2 \\ \hline y_0 & 1 & 0 & 1 \\ y_1 & 1 & 1 & 1 \\ y_2 & 1 & 0 & 1 \end{array}$$

From rule 1 we have,

- y_0 intersections: $(m' = 1, n' = 2)$

$$\{x_0\} \sqsupseteq y_0 \quad \wedge \quad y_0 \sqsubseteq \{z_0, z_2\}$$
$$\rightarrow \quad (z_0 \cap x_0 \neq \phi \wedge z_2 \cap x_0 \neq \phi)$$

- y_1 intersections: $(m' = 1, n' = 3)$

$$\{x_0\} \sqsupseteq y_1 \quad \wedge \quad y_1 \sqsubseteq \{z_0, z_1, z_2\}$$
$$\rightarrow \quad z_0 \cap x_0 \neq \phi \wedge z_1 \cap x_0 \neq \phi \wedge z_2 \cap x_0 \neq \phi$$

- y_2 intersections: $(m' = 3, n' = 2)$

$$\{x_0, x_1, x_2\} \sqsupseteq y_2 \quad \wedge \quad y_2 \sqsubseteq \{z_0, z_2\}$$
$$\rightarrow \quad (x_0 \cap z_0 \neq \phi \vee x_0 \cap z_2 \neq \phi)$$
$$\wedge \quad (x_1 \cap z_0 \neq \phi \vee x_1 \cap z_2 \neq \phi)$$
$$\wedge \quad (x_2 \cap z_0 \neq \phi \vee x_2 \cap z_2 \neq \phi)$$
$$\wedge \quad (z_0 \cap X \neq \phi \;\wedge\; z_2 \cap X \neq \phi)$$

Applying rule 2 we get the following,

- $x_1 \sqsubseteq \{y_2\} \ \wedge \ \{y_2\} \sqsubseteq \{z_0, z_2\} \rightarrow x_1 \cap \{z_1\} = \phi$
- $x_2 \sqsubseteq \{y_2\} \ \wedge \ y_2 \sqsubseteq \{z_0, z_2\} \rightarrow x_2 \cap \{z_1\} = \phi$

Combining the above constraints and selecting the strongest (for example, y_2 intersections gave $x_0 \cap z_0 \neq \phi \vee x_0 \cap z_2 \neq \phi$ and y_0 intersections gave $x_0 \cap z_0 \neq \phi \wedge x_0 \cap z_2 \neq \phi$ and hence the latter is selected), we get the following intersection matrix.

	z_0	z_1	z_2
x_0	1	1	1
x_1	a_1	0	a_2
x_2	b_1	0	b_2

Domain specific constraints must be considered in deriving the physically possible relations which correspond to the result matrix. Table 3.1 represents the mapping between the physically possible relations and the component intersections. The table is compiled from [JB94] where specific constraints were used to prune the physically feasible relations between a region and a line. Each cell in the table contain the relations where there is a non-empty intersection between the corresponding components. If a relation is missing in a cell, then the intersection between the corresponding components is empty. To determine the result of the composition all the propagated constraints in the resulting matrix must be satisfied by carrying out the following steps.

I. For non-empty intersections, get the intersection of the sets of corresponding relations. In the example, $x_0 \cap z_0 \neq \phi \wedge x_0 \cap z_1 \neq \phi \wedge x_0 \cap z_2 \neq \phi$ gives $\{All\} \wedge \{R_1, R_2, R_4, R_{16}, R_{17}, R_{18}, R_{19}\} \cap \{R_1, R_2, R_3, R_4, R_5, R_{13}, R_{14}, R_{15}, R_{16}, R_{17}, R_{18}, R_{19}\} = \{R_1, R_2, R_4, R_{16}, R_{17}, R_{18}, R_{19}\}$.

II. For empty intersections, get the intersection of the complements of the sets of the corresponding relations (or the complement of the union of the set of corresponding relations). In the example, $x_1 \cap z_1 = \phi \wedge x_2 \cap z_1 = \phi$ gives $\{R_1, R_{11}, R_{12}, R_{13}, R_{17}, R_{18}, R_{19}\} \cap$ $\{R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_{15}, R_{16},$ $R_{18}, R_{19}\} = \{R_1, R_{18}, R_{19}\}$.

III. For indefinite intersections, get the union of the sets of corresponding relations. In the example, $x_1 \cap z_0 = a_1 \wedge x_1 \cap z_2 = a_2$ and $x_2 \cap z_0 = b_1 \wedge x_2 \cap z_2 = b_2$ gives $\{All\} \cup \{R_4, R_5, R_6, R_7, R_9, R_{12}, R_{13}, R_{14}, R_{15}, R_{16}, R_{17}, R_{18}, R_{19}\} = \{All\}$ and $\{All\} \cup \{R_7, R_8, R_9, R_{10}, R_{11}, R_{12}, R_{13}, R_{14}, R_{15}, R_{16}, R_{17}, R_{18}\} = \{All\}$

IV. The final required set of relations is then derived by the intersection of all the resulting sets from the previous three steps. In the example, $\{R_1, R_2, R_4, R_{16},$ $R_{17}, R_{18}, R_{19}\} \cap \{R_1, R_{18}, R_{19}\} \cap \{All\} \cap \{All\}$ gives the set $\{R_1, R_{18}, R_{19}\}$ shown in figure 3.11.

This indefinite result is expected according to point II from the analysis where both m' and n' are greater than 1 ($m' = 3, n' = 2$ for y_2 intersections).

The formalism was used to derive the full composition table between two regions and a region and a non-directed line. The full table is given in the appendix. The table shows the conceptual neighbourhood phenomenon observed by Freksa [Fre91a], namely that in the case of indefinite composition the disjunctive set of relations are conceptual neighbours. (Conceptual neighbour relations are created by continuous deformation of one object (shortening or lengthening)).

Note that in the above example a non-directed line is used, with no distinction between the end points. Distinguishing between the end points is necessary if a directed line is used.

	z_0	z_1	z_2
x_0	*All*	$R_1, R_2, R_4, R_{16}, R_{17}, R_{18}, R_{19}$	$R_1, R_2, R_3, R_4, R_5, R_{13}, R_{14}$ $R_{15}, R_{16}, R_{17}, R_{18}, R_{19}$
x_1	*All*	$R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, R_{14}, R_{15}, R_{16}$	$R_4, R_5, R_6, R_7, R_9, R_{12}, R_{13}, R_{14}, R_{15}, R_{16}, R_{17}, R_{18}, R_{19}$
x_2	*All*	$R_9, R_{10}, R_{11}, R_{12}, R_{13}, R_{14}, R_{17}$	$R_7, R_8, R_9, R_{10}, R_{11}, R_{12}, R_{13}, R_{14}, R_{15}, R_{16}, R_{17}, R_{18}$

Table 3.1: Correspondence between the intersection of the components and the relations between a region and a line. Figures for the relations R_1 to R_{19} are shown in the composition tables 3.2 and 3.3.

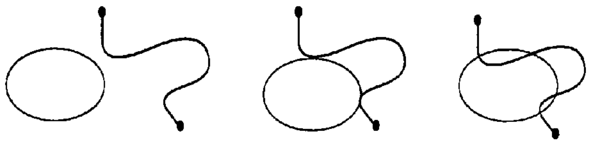
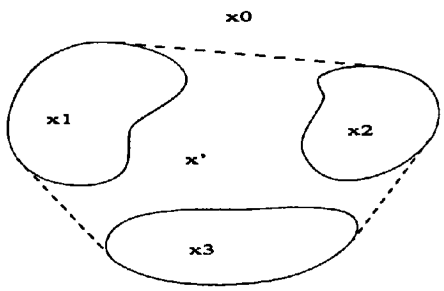


Figure 3.11: Possible relations resulting from the composition in figure 3.10.

3.3 Composite Regions

Composite regions are used here to denote regions with multiple separate components. This type of objects is needed in many application contexts. Consider, for example, modeling a university entity, made of different disconnected buildings in different sites in a city; a country may consist of separate islands, etc. There is a need in spatial databases to model those aggregate objects as wholes, and hence enabling the representation of their relationships in space. This need has been identified in many works [APW$^+$94, HT97]. However, so far, few works addressed this representation problem [CDFC95, NPS97, EGG$^+$99].

One possible method for representing a composite region, proposed here, is by using its convex hull, as shown in figure 3.12(a). The region is defined by the union of its separate component regions as well as their complement which lies within the convex hull, x'. A coarse level of representation is used initially. Further refinement of the object details may be used later for exact determination of relationships.



(a)

x_0				
1	x'			
1	1	x_1		
1	1	0	x_2	
1	1	0	0	x_3

(b)

Figure 3.12: (a) A composite region formed of multiple, separate, parts, x_1, x_2 and x_3, defined using its convex hull. (b) Its corresponding adjacency matrix.

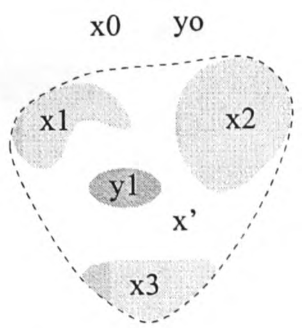
Claramunt [Cla00] addressed the same problem by extending Ladkin's algebra for non-convex intervals into composite regions. This approach can be characterised as holistic. The approach proposed here is compositional where precise intersection relation is defined between each component.

Figure 3.13 shows examples of spatial relationships between composite and simple regions.

3.3.1 Reasoning with Composite Regions

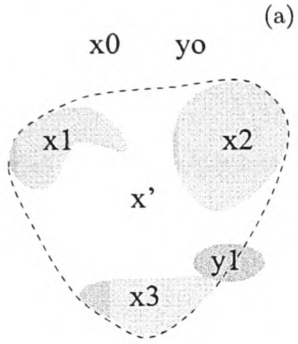
The example in figure 3.14 demonstrates the composition of relations using composite regions. Figure 3.14 shows the relationship between a composite region y and a simple region x in (a) and a concave region z in (b). The intersection matrices for both relations in shown in 3.14(c). Given the relationships in the figure, it is required to derive the possible relationships between x and z.

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,



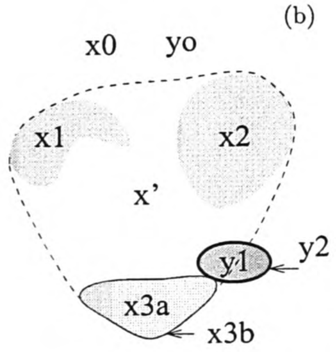
	y_0	y_1
x_0	1	0
$x_{\prime}$	1	1
x_1	1	0
x_2	1	0
x_3	1	0

(a)



	y_0	y_1
x_0	1	1
$x_{\prime}$	1	1
x_1	1	0
x_2	1	0
x_3	1	1

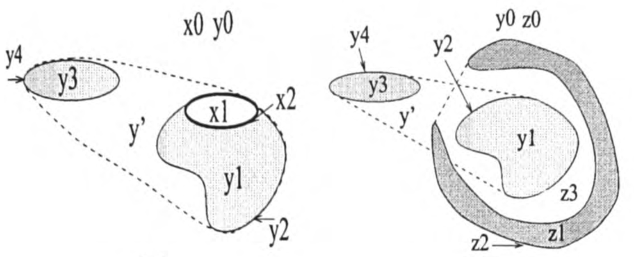
(b)



	y_0	y_1	y_2
x_0	1	1	1
$x_{\prime}$	1	1	1
x_1	1	0	0
x_2	1	0	0
x_{3a}	1	0	0
x_{3b}	1	0	1

(c)

Figure 3.13: (a) Possible relationship between a composite and a simple region. (b) A different relationship, distinguished by y connecting to x_3. (c) The exact nature of relationship is revealed by considering finer object detail.



(a) (b)

x_0	x_2	x_1		z_1	z_2	z_3	z_0
1	1	1	y_1	0	0	1	0
1	1	0	y_2	0	0	1	0
1	0	0	y_3	0	0	0	1
1	0	0	y_4	0	0	0	1
1	0	0	y'	1	1	1	1
1	0	0	y_0	1	1	1	1

(c)

Figure 3.14: (a) and (b) Spatial relationships between different types of regions. (c) Corresponding intersection matrices.

- y_0 intersections:

$$\begin{aligned} \{x_0\} \sqsupseteq y_0 \quad &\wedge \quad y_0 \sqsubseteq \{z_0, z_1, z_2, z_3\} \\ &\rightarrow \quad x_0 \cap z_0 \neq \phi \wedge x_0 \cap z_1 \neq \phi \\ &\wedge \quad x_0 \cap z_2 \neq \phi \wedge x_0 \cap z_3 \neq \phi \end{aligned}$$

- y_1 intersections:

$$\{x_0, x_1, x_2\} \sqsupseteq y_1 \wedge y_1 \sqsubseteq \{z_3\} \rightarrow x_0 \cap z_3 \neq \phi \wedge x_1 \cap z_3 \neq \phi \wedge x_2 \cap z_3 \neq \phi$$

- y_2 intersections:

$$\begin{aligned} \{x_0, x_2\} \sqsupseteq y_2 \quad &\wedge \quad y_2 \sqsubseteq \{z_3\} \\ &\rightarrow \quad x_0 \cap z_3 \neq \phi \wedge x_2 \cap z_3 \neq \phi \end{aligned}$$

- y_3 intersections:

$$\begin{aligned} \{x_0\} \sqsupseteq y_3 \quad &\wedge \quad y_3 \sqsubseteq \{z_0\} \\ &\rightarrow \quad x_0 \cap z_0 \neq \phi \end{aligned}$$

- y_4 intersections:

$$\begin{aligned} \{x_0\} \sqsupseteq y_3 \quad &\wedge \quad y_3 \sqsubseteq \{z_0\} \\ &\rightarrow \quad x_0 \cap z_0 \neq \phi \end{aligned}$$

- y' intersections:

$$\begin{aligned} \{x_0\} \sqsupseteq y_0 \quad &\wedge \quad y_0 \sqsubseteq \{z_0, z_1, z_2, z_3\} \\ &\rightarrow \quad x_0 \cap z_0 \neq \phi \wedge x_0 \cap z_1 \neq \phi \\ &\wedge \quad x_0 \cap z_2 \neq \phi \wedge x_0 \cap z_3 \neq \phi \end{aligned}$$

Note that the intersections of y_0 and y' with both X and Z are identical. The same is true for y_3 and y_4. The observation reduce the number of steps in the composition by a third. Applying rule 2 we get the following,

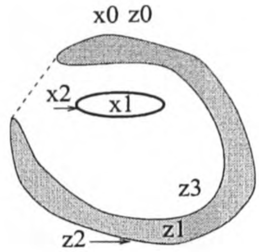
- $x_1 \sqsubseteq y_1 \;\wedge\; y_1 \sqsubseteq z_3 \rightarrow x_1 \cap z_1 = \phi \wedge x_1 \cap z_2 = \phi \wedge x_1 \cap z_0 = \phi$
- $x_2 \sqsubseteq \{y_1, y_2\} \;\wedge\; \{y_1, y_2\} \sqsubseteq \{z_3\} \rightarrow x_2 \cap z_1 = \phi \wedge x_2 \cap z_2 = \phi \wedge x_2 \cap z_0 = \phi$
- $z_1 \sqsubseteq \{y', y_0\} \;\wedge\; \{y', y_0\} \sqsubseteq \{x_0\} \rightarrow z_1 \cap x_1 = \phi \wedge z_1 \cap x_2 = \phi$
- $z_0 \sqsubseteq \{y_3, y_4, y', y_0\} \;\wedge\; \{y_3, y_4, y', y_0\} \sqsubseteq \{x_0\} \rightarrow z_0 \cap x_1 = \phi \wedge z_0 \cap x_2 = \phi$
- $z_2 \sqsubseteq \{y', y_0\} \;\wedge\; \{y', y_0\} \sqsubseteq \{x_0\} \rightarrow z_2 \cap x_1 = \phi \wedge z_2 \cap x_2 = \phi$

Refining the above constraints, we get the intersection matrix in figure 3.15(a) which maps to one definite relation 3.15(b).

A different conclusion is obtained if the relationship between objects x and y is as shown in figure 3.16(a). Their corresponding intersection matrix is in 3.16(b). The composition of the relationships between x, y and z in this case will result in the indefinite matrix in figure 3.16(d) which corresponds to one of the three possible relations in 3.16(e).

	z_1	z_2	z_3	z_0
x_1	0	0	1	0
x_2	0	0	1	0
x_0	1	1	1	1

(a)



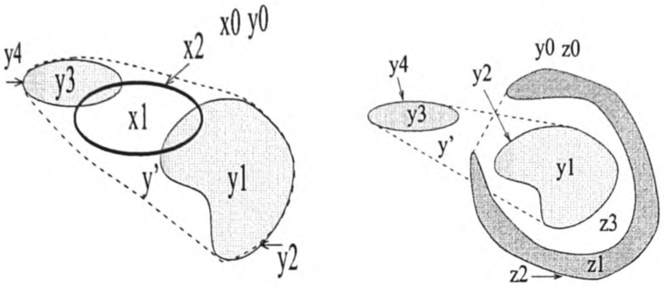
(b)

Figure 3.15: (a) Resulting intersection matrix for the composition in figure 3.14 (b) Its corresponding definite relation.

3.4 Conclusions

A general approach for spatial reasoning is proposed. The approach consists of a set of two general constraints to govern the spatial relationships between objects in space and two general rules to propagate two definite relationships between objects in space. The formalism is based on representing the objects and their space topology by an adjacency matrix where a partition strategy of objects and space is carried out to reflect the specific decomposition of interest in different applications. The topological relations are then represented by an intersection matrix between different objects and space parts. The adjacency matrix can represent the topology of complex objects and collections of objects such as whole geographic maps.

The reasoning process is proved to be general and is shown to be applicable on objects of arbitrary complexity and of different dimensions. The simplicity of the approach is based on using two reasoning rules for propagating empty and non-empty intersections. The rules allows for the derivation of the different composition tables, an essential task for developing a general spatial reasoning mechanism. An analysis of the reasoning process revealed important and interesting features for deriving definite and indefinite spatial relations. The formalism is flexible in that it allows the definition and use of domain specific



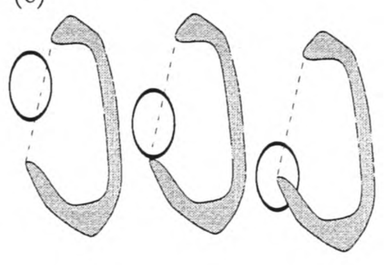
(a) (b)

x_0	x_2	x_1		z_1	z_2	z_3	z_0
1	1	1	y_1	0	0	1	0
1	1	1	y_2	0	0	1	0
1	1	1	y_3	0	0	0	1
1	1	1	y_4	0	0	0	1
1	1	1	y'	1	1	1	1
1	0	0	y_0	1	1	1	1

(c)

	z_1	z_2	z_3	z_0
x_1	?	?	1	0
x_2	?	?	1	0
x_0	1	1	1	1

(d)



(e)

Figure 3.16: (a), (b) Example relationships between objects x, y and z. (c) Corresponding intersection matrix. (d) Intersection matrix resulting from the composition of the two relationships. (e) Possible relations of the resulting composition.

constraints according to the application studied. A major advantage of the method is that reasoning between objects of any complexity can be achieved in a number of defined and limited steps. A prototype for the reasoning formalism was implemented and described in appendix A.

	disjoint(x, y)	*meet*(x, y)	*inside*(x, y)	*coverdBy*(x, y)	*contain*(x, y)	*cover*(x, y)	*overlap*(x, y)
$R_1(y, z)$	*all*	*all*	1	1	*all*	*all*	*all*
$R_2(y, z)$	1, 2, 4, 16, 17, 18, 19	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19	1	1, 2	9, 10, 11, 12, 13, 14, 17,	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17	*all*
$R_3(y, z)$	1, 18, 19	1, 2, 3, 4, 5, 6, 7, 8, 15, 16, 18, 19	1	1, 2, 3,	11, 12, 13	3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	*all*
$R_4(y, z)$	1, 2, 4, 16, 17, 18, 19	1, 2, 3, 4, 5, 6, 7, 9, 14, 15, 16, 17, 18, 19	1	1, 2, 4, 19,	9, 10, 11, 12, 13, 14, 17	4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17	*all*
$R_5(y, z)$	1, 18, 19,	1, 2, 3, 4, 5, 6, 7, 15, 16, 18, 19	1	1, 2, 3, 4, 5, 19	11, 12, 13	5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	*all*
$R_6(y, z)$	1	1, 2, 3, 4, 5, 6, 19	1	1, 2, 3, 4, 5, 6, 19	11	6, 7, 8, 9, 10, 11, 12	*all*
$R_7(y, z)$	1	1, 2, 3, 4, 5, 19	1, 18, 19	1, 2, 3, 4, 5, 6, 7, 15, 16, 18, 19	11	7, 8, 9, 10, 11, 12	*all*
$R_8(y, z)$	1	1, 2, 3	1, 18, 19	1, 2, 3, 4, 5, 6, 7, 8, 15, 16, 18, 19	11	8, 10, 11	*all*
$R_9(y, z)$	1	1, 2, 4, 19	1, 2, 4, 16, 17, 18, 19	1, 2, 3, 4, 5, 6, 7, 9, 14, 15, 16, 17, 18, 19	11	9, 10, 11, 12	*all*

Table 3.2: Part of the composition table between two regions and a region and a line. The numbers in the table correspond to relations R_1 to R_{19}. Cell entries in the table represent the result of composing the corresponding relationships.

	disjoint(x, y)	*meet*(x, y)	*inside*(x, y)	*coverdBy*(x, y)	*contain*(x, y)	*cover*(x, y)	*overlap*(x, y)
$R_{10}(y, z)$	1	1, 2	1, 2, 4, 16, 17, 18, 19	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19	11	10, 11	*all*
$R_{11}(y, z)$	1	1	*all*	*all*	11	11	*all*
$R_{12}(y, z)$	1	1, 19	1, 2, 3, 4, 5, 13, 14, 15, 16, 17 18, 19	1, 2, 3, 4, 5, 6, 7, 9, 12, 13, 14, 15, 16, 17, 18, 19	11	11, 12	*all*
$R_{13}(y, z)$	1, 18, 19	1, 18, 19	1, 2, 3, 4, 5, 13, 14, 15, 16, 17, 18, 19	1, 2, 3, 4, 5, 13, 14, 15, 16, 17, 18, 19	11, 12, 13	11, 12, 13	*all*
$R_{14}(y, z)$	1, 18, 19	1, 2, 4, 16, 18, 19	1, 2, 4, 16, 17, 18, 19	1, 2, 3, 4, 5 14, 15, 16, 17, 18, 19	11, 12, 13	9, 10, 11, 12, 13, 14	*all*
$R_{15}(y, z)$	1, 18, 19	1, 2, 3, 4, 5, 15, 16, 18, 19	1, 18, 19	1, 2, 3, 4, 5, 15, 16, 18, 19	11, 12, 13,	7, 8, 9, 10, 11, 12, 13, 14, 15	*all*
$R_{16}(y, z)$	1, 2, 4, 16, 17, 18, 19	1, 2, 3, 4, 5, 14, 15, 16, 17, 18, 19	1, 18, 19	1, 2, 4, 16, 18, 19	9, 10, 11, 12, 13, 14, 17	7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17	*all*
$R_{17}(y, z)$	1, 2, 4, 16, 17, 18, 19	1, 2, 4, 16, 17, 18, 19	1, 2, 4, 16, 17, 18, 19	1, 2, 4, 16, 17, 18, 19	9, 10, 11, 12, 13, 14, 17	9, 10, 11, 12, 13, 14, 17	*all*
$R_{18}(y, z)$	1, 2, 3, 4, 5, 13, 14, 15, 16, 17, 18, 19	1, 2, 3, 4, 5, 13, 14, 15, 16, 17, 18, 19	1, 18, 19	1, 18, 19	7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	*all*
$R_{19}(y, z)$	1, 2, 3, 4, 5, 13, 14, 15, 16, 17, 18, 19	1, 2, 3, 4, 5, 6, 7, 9, 12, 13, 14, 15, 16, 17, 18, 19	1	1, 19	7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19	*all*

Table 3.3: The rest of the composition table between two regions and a region and a line.

Chapter 4

Handling Orientation and Proximity and Time

In this chapter, the representation and reasoning formalism is applied on the two other types of qualitative relationships, namely, orientation or direction relationships and proximity relationships.

4.1 Representation of Orientation Relations

On the orientation axis, two types of rotations can be distinguished: the rotation of the object around the reference object and the rotation of the object around itself. The rotation of an object around another changes its directional position with respect to that object, e.g. the object changes from being in-front (or east) to being to the left (or north) and so on. On the other hand, the rotation of the object around itself changes the orientation between the frames of reference of the two objects and the position of the reference object with respect to the referenced one. For example, objects changing from being front-to-front to being front-to-back or front-to-side and so on. Note that in the later case, the relative cardinal directional position remains constant, i.e. the object changes from being front-facing to back-facing while still being to the north or to the south.

The above two rotations reflects two types of frames of reference in the orientation space which in turn produces two types of orientation relations [RS88]:

Extrinsic: when a fixed external frame of reference is used for both the object spaces, for example cardinal direction orientation (east, west, north, south) as shown in figure 4.1.

Intrinsic (Body orientation): when each object carries its own frame of reference determined by some inherent property in that object, for example, front of the house. Typical values on this reference frame are *front, back, left, right* as shown in figure 4.1.

Note that since in the intrinsic frame of reference the rotation of the objects around itself affects its relative orientation, both relations of the objects are needed to fully represent the relation. For example, the relationship "the car is `in-front` of the house", does not imply that "the house is `behind` the car", nor does it imply any other relation. In the extrinsic frame of reference only one relation is enough.

Similar to the representation strategy used in the previous chapter for topological relations, the adjacency between the object and the semi-infinite orientation areas are explicitly represented for each object. Orientation relations between two objects are then represented by the intersection of the components of their object spaces.

The object divides the space into semi-infinite areas, denoted *orientation areas*, (minimum of two areas, such as *front*, *back*),

Several schemes exist for the division of space to represent areas of acceptance for each orientation such as conical or rectangular. The approach defined here is independent of the scheme used to divide the space. Hence, the formalism is expressive and can accommodate to the scheme of relation representation. Space divisions used in this thesis are chosen for clarity and readability. More complex divisions should be treated in a similar way.

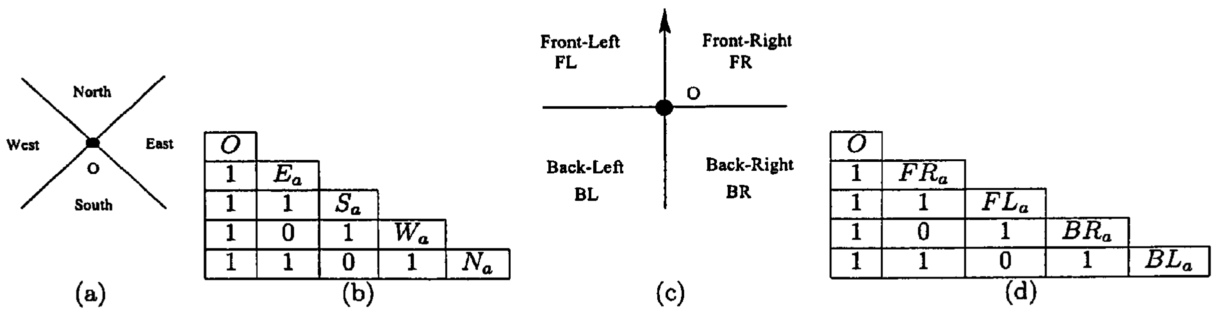


Figure 4.1: (a) Extrinsic frame of reference and its corresponding adjacency structure in (b). (c) Extrinsic frame of reference and its corresponding adjacency structure in (d).

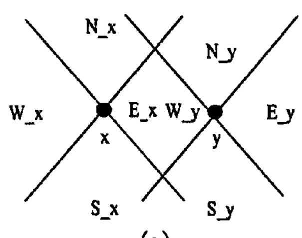
The Representation of Orientation Relations

The relations are represented through the intersection of the space components in which the two objects are embedded.

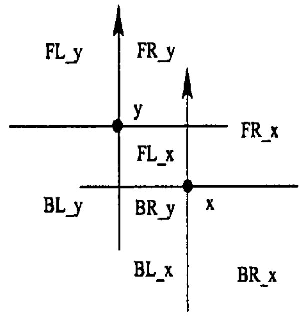
In the orientation space, the complement of the objects, viz. x_0 is divided into a number of semi-infinite areas according to the granularity of relations required. For simplicity, a granularity of four is used in the following examples for both the extrinsic and intrinsic cases, however the methodology is valid for any required granularity. Note that for simplicity object shapes are approximated by points in the examples. The formalism can handle different types of objects and space divisions as long as the resulting embedding space is dense and connected. However, in the orientation space, objects are usually approximated by points or by their bounding boxes.

Examples:

In figure 4.2(a) an extrinsic orientation relation is shown and the corresponding adjacency matrix in given in (b). In 4.2(c) objects with an intrinsic frame of reference are shown. The components of spaces X and Y are as follows: $X = x \cup FR_x \cup FL_x \cup BR_x \cup BL_x, Y = y \cup FR_y \cup FL_y \cup BR_y \cup BL_y$ where FR_i, FL_i, BR_i, BL_i denote the orientation relations: *Front-Right*, *Front-Left*, *Back-Right* and *Back-Left* respectively. The intersection matrix



(a)



(c)

	E_x	S_x	W_x	N_x
E_y	1	0	0	0
S_y	1	1	0	0
W_y	1	1	1	1
N_y	1	0	0	1

(b)

	x	FR_x	FL_x	BR_x	BL_x
y	0	0	1	0	0
FR_y	0	1	1	0	0
FL_y	0	0	1	0	0
BR_y	1	1	1	1	1
BL_y	0	0	1	0	1

(d)

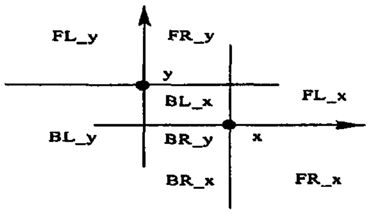
Figure 4.2: (a) Example of cardinal directions and its corresponding intersection matrix in (b). (b) Example of an intrinsic orientation relation and its corresponding intersection matrix in (d). The arrow on the figure denotes the front of the object.

corresponding to this relation is shown in figure 4.2(d).

As mentioned earlier both the relationship and its converse are needed to completely define the orientation relation in the case of the intrinsic frame of reference. For example, in figure 4.2(c), the relationship between objects x and y is defined by $BR(x, y) \wedge FL(y, x)$. If either of the objects rotates around itself, its relative relationship with the other object shall change as well, as shown in figure 4.3. In the figure, object x has changed its orientation and hence also changed its relationship with object y to be: $BR(x, y) \wedge BL(y, x)$.

4.1.1 Reasoning over the Orientation Space

In this section, the reasoning formalism developed for topological relations is extended and applied for reasoning over orientation relations.



(a)

	x	FR_x	FL_x	BR_x	BL_x
y	0	0	0	0	1
FR_y	0	0	1	0	1
FL_y	0	0	0	0	1
BR_y	1	1	1	1	1
BL_y	0	0	0	1	1

(b)

Figure 4.3: (a) Changing the body orientation of object x gives a different relationship defined by the matrix in (b).

For the sake of simplicity, the objects and the bounding lines of the orientations areas are omitted. This does not affect the reasoning process in the examples given or the features of the formalism since both general constraints are preserved for the semi-infinite areas. A mapping between non-empty intersections of space and the corresponding possible relations is given in figure 4.4.

Each cell in the table contain the relations where there is a non-empty intersection between the corresponding components. If a relation is missing in a cell, then the intersection between the corresponding components is empty for the missing relation. For example, the highlighted cell in the table corresponding to the components FR_x and FR_z is interpreted as follows: if we know that the intersection of the components FR_x and FR_z is not empty, then the relation between objects x and z could be either of the following:

a. $FR(z,x) \wedge (BL(x,z) \vee BR(x,z) \vee FL(x,z) \vee FR(x,z))$. Another way of expressing this is: $FR(z,x) \wedge All(x,z)$, or,

b. $BR(z,x) \wedge (FL(x,z) \vee FR(x,z))$, or,

c. $BL(z,x) \wedge FR(x,z)$, or,

d. $FL(z,x) \wedge (BR(x,z) \vee FR(x,z))$

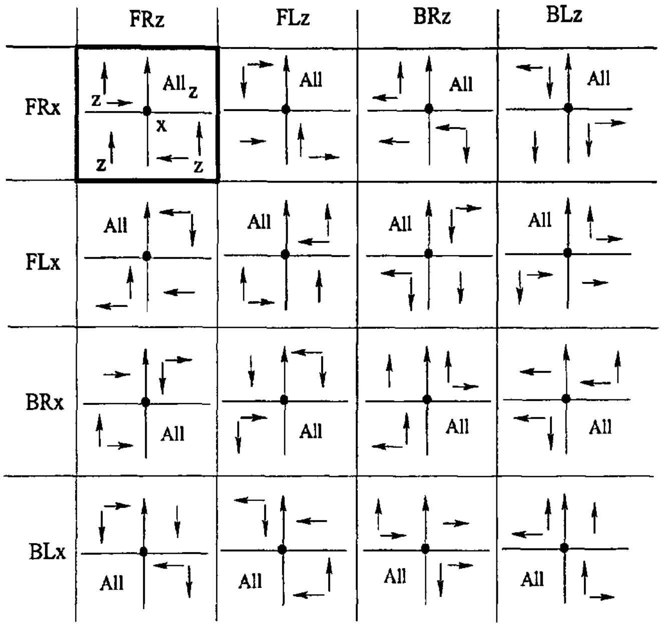


Figure 4.4: Correspondence between the intersection of the components and the relations in the intrinsic frame of reference. The highlighted cell entry is explained in the text. The cross represents the space of x, and the small arrows represent the front direction of object z.

Middle Object Reasoning:

The above table can also be used for reasoning with partial knowledge of the relations between the middle object and the reference objects. For example, if the only knowledge available is : $FR(y,x) \wedge FR(y,z)$, the the relations between x and z are the same as the set listed above. This is due to that composition of the relations $FR(y,x)$ and $FR(y,z)$ gives the following result: $\{FR_x\} \sqsupset y \sqsubset \{FR_z\} \rightarrow FR_x \cap FR_z \neq \phi$.

Example: Propagation of Definite Compositions

Consider the simple example of composing the relationships: $FL(y,x) \wedge FL(x,y) \wedge BR(y,z) \wedge BR(z,y)$. The relationships and their corresponding intersection matrices are shown in figure 4.5(a) and (b).

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- FR_y intersections:

$$\begin{aligned} \{FL_x, BL_x\} \sqsupseteq FR_y \sqsubseteq \{BR_z, BL_z\} \quad &\rightarrow \quad (FL_x \cap BR_z \neq \phi \vee FL_x \cap BL_z \neq \phi) \\ &\wedge \quad (BL_x \cap BR_x \neq \phi \vee BL_x \cap BL_z \neq \phi) \end{aligned}$$

- FL_y intersections:

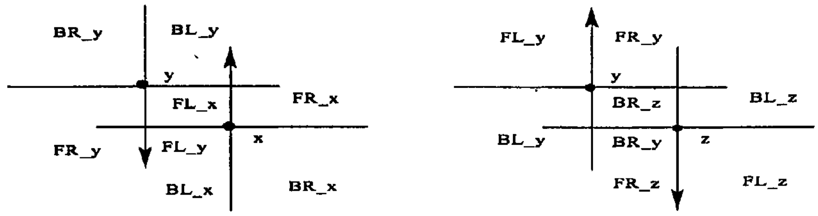
$$\{X\} \sqsupseteq FL_y \sqsubseteq \{BR_z\} \quad \rightarrow \quad (BR_z \cap \{X\} \neq \phi)$$

  Note that the result of this composition can only identify the relative position of x to z $(BR(x,z))$, but not vice versa.

- BR_y intersections:

$$\{FL_x\} \sqsupseteq BR_y \sqsubseteq \{Z\} \quad \rightarrow \quad (FL_x \cap \{Z\} \neq \phi)$$

  From this constraint it can be deduced that the relation between z and x is $FL(z,x)$.



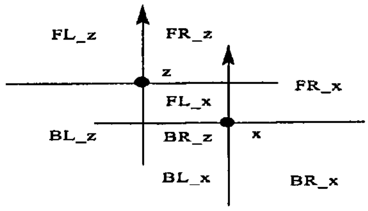
(a)

FR_x	FL_x	BR_x	BL_x		FR_z	FL_z	BR_z	BL_z
0	1	0	1	FR_y	0	0	1	1
1	1	1	1	FL_y	0	0	1	0
0	1	0	0	BR_y	1	1	1	1
1	1	0	0	BL_y	1	0	1	0

(b)

	FR_x	FL_x	BR_x	BL_x
FR_z	1	1	0	0
FL_z	0	1	0	0
BR_z	1	1	1	1
BL_z	0	1	0	1

(c)



(d)

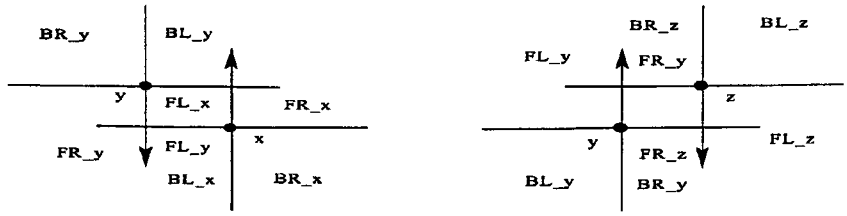
Figure 4.5: (a) Composing the relationships $FL(x, y) \wedge FL(y, x)$ and $BR(y, z) \wedge BR(z, y)$. (c) Corresponding intersection matrices. (d) Resulting propagated constraints. (e) Corresponding (definite) relationship.

- BL_y intersections:

$$\begin{aligned} \{FR_x, FL_x\} \sqsupseteq BL_y \sqsubseteq \{FR_z, BR_z\} \quad &\rightarrow \quad (FR_x \cap FR_z \neq \phi \vee FR_x \cap BR_z \neq \phi) \\ &\wedge \quad (FL_x \cap FR_z \neq \phi \vee FL_x \cap BR_z \neq \phi) \end{aligned}$$

Note the intersections of the components FL_y and BR_y have fully identified the composed relation, namely, $BR(x, z) \wedge FL(z, x)$. In this case, we don't need to apply rule 2. However for completeness the propagation of constraints by rule 2 are as follows:

- $\{FR_x\} \sqsubseteq (FL_y \cup BL_y) \sqsubseteq \{BR_z, FR_z\} \rightarrow FR_x \cap \{FL_z, BL_z\} = \phi$



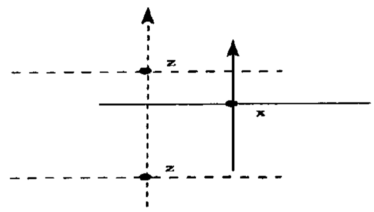
(a)

FR_x	FL_x	BR_x	BL_x		FR_z	FL_z	BR_z	BL_z
0	1	0	1	FR_y	1	1	1	1
1	1	1	1	FL_y	1	0	1	0
0	1	0	0	BR_y	1	1	0	0
1	1	0	0	BL_y	1	0	0	0

(b)

	FR_z	FL_z	BR_z	BL_z
FR_x	1, b_1	0	b_1	0
FL_x	a_1, b_2, 1	a_2, 1	a_3,b_2	a_4
BR_x	b_3	0	b_3	0
BL_x	a_1, b_4	a_2	b_4, a_3	a_4

(c)



(d)

Figure 4.6: (a) Composing the relationships $FL(x,y) \wedge FL(y,x)$ and $FR(y,z) \wedge FR(z,y)$. (b) Corresponding intersection matrices. (c) Resulting propagated constraints. (d) Corresponding (indefinite) relationships.

- $\{BR_x\} \sqsubseteq FL_y \sqsubseteq \{BR_z\} \rightarrow BR_x \cap \{FR_z, FL_z, BL_z\} = \phi$
- $\{BL_x\} \sqsubseteq \{FL_y, FR_y\} \sqsubseteq \{BR_z, BL_z\} \rightarrow BL_x \cap \{FR_z, FL_z\} = \phi$
- FL_x has no empty intersections since $l' = l$.

Grouping the above constraints, we get the intersection matrix in figure 4.5(c). Using table 4.4, it can be seen that the result matrix corresponds to the relationships $BR(x,z) \wedge FL(z,x)$ as in figure 4.5(d).

Example: Propagation of Indefinite Compositions

Consider the relationships in figure 4.6: $FL(y,x) \wedge FL(x,y) \wedge FR(z,y) \wedge FR(y,z)$. The corresponding intersection matrices are shown in (b). The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- FR_y intersections:

$$
\begin{aligned}
\{Z\} \sqsupseteq FR_y \sqsubseteq \{FL_x, BL_x\} \quad &\rightarrow \quad (FR_z \cap FL_x \neq \phi \vee FR_z \cap BL_x \neq \phi) && (a1)\\
&\wedge \quad (FL_z \cap FL_x \neq \phi \vee FL_z \cap BL_x \neq \phi) && (a2)\\
&\wedge \quad (BR_z \cap FL_x \neq \phi \vee BR_z \cap BL_x \neq \phi) && (a3)\\
&\wedge \quad (BL_z \cap FL_x \neq \phi \vee BL_z \cap BL_x \neq \phi) && (a4)
\end{aligned}
$$

- FL_y intersections:

$$
\begin{aligned}
\{FR_z, BR_z\} \sqsupseteq FL_y \sqsubseteq \{X\} \quad &\rightarrow \quad (FR_x \cap FR_z \neq \phi \vee FR_x \cap BR_z \neq \phi) && (b1)\\
&\wedge \quad (FL_x \cap FR_z \neq \phi \vee FL_x \cap BR_z \neq \phi) && (b2)\\
&\wedge \quad (BR_x \cap FR_z \neq \phi \vee BR_x \cap BR_z \neq \phi) && (b2)\\
&\wedge \quad (BL_x \cap FR_z \neq \phi \vee BL_x \cap BR_z \neq \phi) && (b2)
\end{aligned}
$$

- BR_y intersections:

$$\{FR_z, FL_z\} \sqsupseteq BR_y \sqsubseteq \{FL_x\} \quad \rightarrow \quad (FL_x \cap FR_z \neq \phi \wedge FL_x \cap FL_z \neq \phi)$$

- BL_y intersections:

$$\{FR_z\} \sqsupseteq BL_y \sqsubseteq \{FR_x, FL_x\} \quad \rightarrow \quad (FR_z \cap FR_x \neq \phi \wedge FR_z \cap FL_x \neq \phi)$$

Applying rule 2 we get the following,

- $FL_z \sqsubseteq \{FR_y \cup BR_y\} \sqsubseteq \{FL_x, BL_x\} \rightarrow FL_z \cap FR_z = \phi \wedge FL_z FL_z \cap BR_x = \phi$

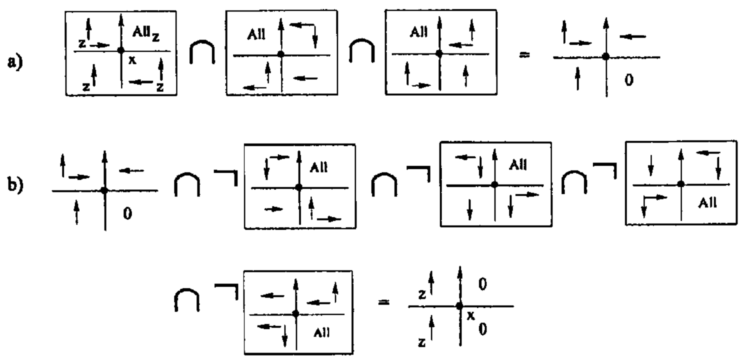


Figure 4.7: The process of mapping the constraints propagated by the reasoning rules to the set of possible relations, namely, $(FR(x,z) \wedge BL(z,x)) \vee (BR(x,z) \wedge FL(z,x))$. The figure is explained in the text.

- $BL_z \sqsubseteq FR_y \sqsubseteq \{FL_x, BL_x\} \rightarrow BL_z \cap FR_x = \phi \wedge BL_z \cap BR_x = \phi$

Refining the above constraints, we get the intersection matrix in figure 4.6(c). Using the table 4.4, we get the possible relations in figure 4.6(d). Note that the conditions: $(a_1), (a_2), (b_1)$ and (b_2) are satisfied by definite intersections. The process of mapping the propagated intersections into possible relations in the table is carried out by finding the intersection of the set of relations corresponding to cells of value 1 in the matrix with the complement of the set of relations corresponding to cells of value 0 in the matrix. This process is demonstrated in figure 4.7. In 4.7(a) the intersection of the set of the relations corresponding to cells of value 1 is shown and in (b) the result from (a) is intersected with the complements of the sets of relations corresponding to cells of value 0.

The result of the composition is indefinite with ambiguity in the relative positions of the objects. The possible resulting relations between objects x and z are: $(FR(x,z) \wedge BL(z,x)) \vee (BR(x,z) \wedge FL(z,x))$

<u>**Example: Propagation of the Universal Relation (no information)**</u>

Consider the relationships in figure 4.8: $east(y,x) \wedge west(z,y)$. The corresponding intersection matrices are shown in figure.

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- E_y intersections:

$$\{E_z\} \sqsupseteq E_y \sqsubseteq \{E_z\} \quad \rightarrow \quad E_x \cap E_z \neq \phi$$

- S_y intersections:

$$\begin{aligned} \{E_z, S_z\} \sqsupseteq S_y \sqsubseteq \{E_x, S_x\} \quad &\rightarrow \quad (E_z \cap E_x \neq \phi \vee E_z \cap S_x \neq \phi) \\ &\wedge \quad (S_z \cap E_x \neq \phi \vee S_z \cap S_x \neq \phi) \end{aligned}$$

  The above two constraints are satisfied as an implication of the first general constraint, since $E_z \cap E_x \neq \phi$ and $S_z \cap S_x \neq \phi$.

- W_y intersections:

$$\{X\} \sqsupseteq W_y \sqsubseteq \{Z\} \quad \rightarrow \quad \{X\} \cap \{Z\} \neq \phi$$

  which is a propagation of the second general constraint.

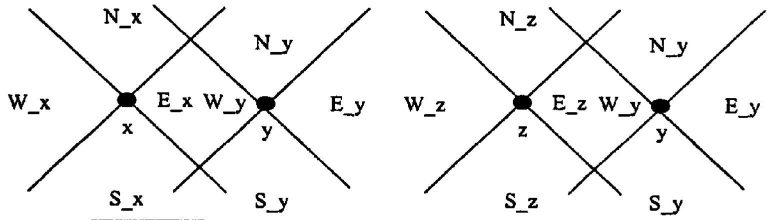
- N_y intersections:

$$\begin{aligned} \{E_z, N_z\} \sqsupseteq N_y \sqsubseteq \{E_x, N_x\} \quad &\rightarrow \quad (E_z \cap E_x \neq \phi \vee E_z \cap N_x \neq \phi) \\ &\wedge \quad (N_z \cap E_x \neq \phi \vee N_z \cap N_x \neq \phi) \end{aligned}$$

  The above two constraints are satisfied as an implication of the first general constraint, since $E_z \cap E_x \neq \phi$ and $N_z \cap N_x \neq \phi$.

The above intersections have all propagated implications of the general constraints, i.e. the composition of the relations does not provide any definite intersections and results in an indefinite relations of: $E(x,z) \vee S(x,z) \vee W(x,z) \vee N(x,z)$.

In applying rule 2, since $m' = m \wedge n' = n$ for the component W_y, no empty intersection can be propagated.



E_x	S_x	W_x	N_x		E_z	S_z	W_z	N_z
1	0	0	0	E_y	I	0	0	0
1	1	0	0	S_y	1	1	0	0
1	1	1	1	W_y	1	1	1	1
1	0	0	1	N_y	1	0	0	1

(c)

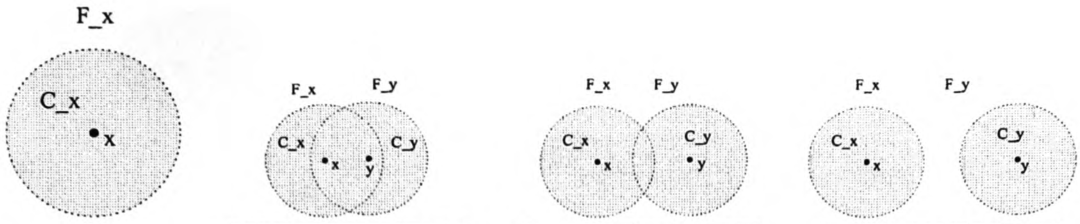
Figure 4.8: Composing cardinal relations resulting where no information is propagated.

A similar example in the case of the intrinsic frame of reference is by composing the relations: $FL(y,x) \wedge FL(x,y) \wedge FL(z,y)$ and $R(y,z)$. In this case the orientation of the frames of reference of objects x and z can be inferred, but not their relative position.

4.2 Representation and Reasoning over Proximity Relations

Proximity is a fuzzy relationship which is context dependent. (A place may be far if a person is walking, when it is relatively near if he is making the trip by car). Consider for example the following relationships: *I am standing near the desk* and *Bristol is near Cardiff*. The *near* relationship conveys a different measure of distance in both cases. Thus these type of relationships would apply only to groups of objects on the same "resolution" level and would not be used across levels. Different granularities of proximity relations can be defined on different levels and according to the application considered. Proximity is also a fuzzy concept, and that is why it is normally associated in human speech with adjectives such as, almost, very, nearly, approximately or quite.

The space around the reference objects is divided into concentric circles defining the areas



(a)

	y	C_y	F_y
x	0	1	0
C_x	1	1	1
F_x	0	1	1

(b)

	y	C_y	F_y
x	0	0	1
C_x	0	1	1
F_x	1	1	1

(c)

	y	C_y	F_y
x	0	0	1
C_x	0	0	1
F_x	1	1	1

(d)

Figure 4.9: (a) Simple division in the proximity space: close denoted by C_x and far denoted by F_x. Possible relationships in this space (b) close(x,y), (c) semi-close(x,y), (d) far(x,y).

for different proximity relations such as, *very close*, *close*, *far*, *very far*, $\cdots$. On the lowest resolution, only two relationships can be distinguished namely, *close* and *far*, defined by two regions around the object separated by a circle around the object.

In general, reasoning about proximity relationships are always associated with reasoning about orientation relationships. In this section, it is shown how reasoning over proximity relations only can propagate useful information.

The representation formalism described earlier is used for the definition of the proximity space as shown in figure 4.9(a). The figure illustrates simple proximity spaces for objects x and y where the space is defined in two regions: a finite acceptance area defining the relationships *close* and a semi-infinite acceptance area defining the relationship far. Possible proximity relationships between x and y are defined by the combinatorial combination of the intersection components. Three relationships are possible in this case. These are shown along with their corresponding intersection matrices in figure 4.9.

Note that even though our initial proximity spaces defined only two proximity regions of *close* and *far*, a third relationship was distinguished in the interaction between the spaces

x	C_x	F_x		z	C_z	F_z
0	1	0	y	0	1	0
1	1	1	C_y	1	1	1
0	1	1	F_y	0	1	1

(a) (b)

Figure 4.10: Composing relations $close(x, y) \wedge close(y, z)$ in (a). (b) their corresponding intersection matrices.

as shown in figure 4.9(c). This relationship is defined when the region c_x has an empty intersection with y and a non-empty intersection with c_y and vice versa. If the relations $close(x, y)$ implies that y lies within a distance rad_c from x, where rad_c is the radius of the *close* region, $(0 < distance(x, y) \leq rad_c)$, then the relationship in figure 4.9(c) defines the region $rad_c < distance(x, y) \leq 2 * rad_c$ and is denoted $semi - close$.

The production of the later relation is an implication of a domain specific constraint in the proximity space where the region around the object are usually defined as circular buffers of constant radii. This relation is usually recognised when three objects are considered and one of them is close to both the other two.

4.2.1 Reasoning over the Proximity Space

The general constraints and the reasoning rules are applicable in the proximity space as illustrated in the following example.

Example

Consider the relationships in figure 4.10: $close(x, y) \wedge close(y, z)$ The corresponding intersection matrices are shown in figure.

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- y intersections: $(m' = 1 \wedge n' = 1)$

$$\{C_x\} \sqsupseteq y \sqsubseteq \{C_z\} \quad \rightarrow \quad C_x \cap C_z \neq \phi$$

- C_y intersections: $(m' = m \wedge n' = n)$

$$\{X\} \sqsupseteq C_y \sqsubseteq \{Z\} \quad \rightarrow \quad X \cap Z \neq \phi$$

  which is an implication of the second general constraint.

- F_y intersections:

$$\begin{aligned} \{C_x, F_x\} \sqsupseteq F_y \sqsubseteq \{C_z, F_z\} \quad &\rightarrow \quad (C_x \cap C_z \neq \phi \vee C_x \cap F_z \neq \phi) \qquad (a) \\ &\wedge \quad (F_x \cap C_z \neq \phi \vee F_x \cap F_z \neq \phi) \qquad (b) \end{aligned}$$

  Condition (a) is already satisfied from the intersection of y and condition (b) is an application of the first general constraint, $F_x \cap F_z \neq \phi$.

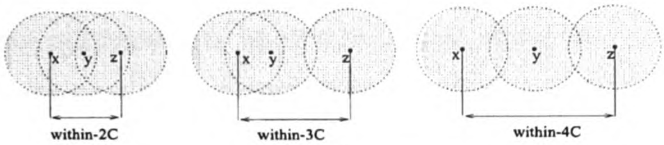
Refining the above constraints, we get the following resulting intersection matrix:

	z	C_z	F_z
x	?	?	?
C_x	?	1	?
F_x	?	?	1

The matrix corresponds to either of the relations in figure 4.9(b) and (c). Hence the composition produces an indefinite result: $close(c, y) \wedge close(y, z) \rightarrow close(x, z) \vee semi-close(x, z)$. The result can also be expressed in a quantitative distance measure as: $close(c, y) \wedge close(y, z) \rightarrow 0 < distance(x, z) \leq 2 * rad_c$. This relation is denoted *within-2c* and read within twice the close distance.

In a similar way, other distance measures can be defined by multiples of distances to get finer granularity relations. Some examples of this relations are shown in figure 4.11. A composition table for the set of relations in figure 4.9 is given in 4.11(b).

Note that composing orientation and proximity relations would give more precise results. Heterogeneous reasoning is out of the scope of this chapter and is considered later on.

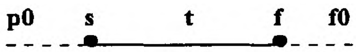


(a)

	$C(y,z)$	$SC(y,z)$	$F(y,z)$
$C(x,y)$	$within-2c(x,z)$	$within-3c(x,z)$	U
$SC(x,y)$	$within-3c(x,z)$	$within-4c(x,z)$	U
$F(x,y)$	U	U	U

(b)

Figure 4.11: (a) Examples of some distance measures defining proximity relationships. (b) Composition table for the relationships in figure 4.9.



f_0				
1	f			
0	1	t		
0	0	1	s	
0	0	0	1	p_0

Figure 4.12: a) An event, b) representation of the event by adjacency matrix.

4.3 Representation and Reasoning in the Temporal Domain

One of the important research areas currently investigated is the common aspects of reasoning about space and time [CDF97]. Towards this aim, in this section it is shown how the formalism can be applied to temporal intervals. The reasoning formalism can be applied to order relations by considering a 1D space where the object (or value) divides that space into two semi-infinite lines, one representing all objects (values) with the relation $<$ and the other for values $>$. In the temporal domain (an order domain), consider an event e in an event space E as shown in figure 4.12. e can be decomposed into the following components: s: its start, f: its finish, t: its duration. The event space E is composed of e and p_0: a semi-infinite line representing the past of e and f_0: a semi infinite line representing the future of e. The connectivity matrix for E is as shown in figure 4.12(b).

The relationship between two events can be represented by an intersection matrix. For example the overlap relationship in figure 4.13 can be represented by the matrix in the same figure. Both the general space constraints in section 1.3 are also applicable in the temporal domain, $f_{01} \cap f_{02} \neq \phi$ and $p_{01} \cap p_{02} \neq \phi$, i.e. the future as well as the past of any two events must intersect.

The analysis of indefinite and definite intersections given earlier is also applicable here. For example, if the two relations $during(A, B)$ and $during(C, B)$ are composed, then all the components of interval B can intersect only with the futures or pasts of both intervals A and C or with every component of the intervals A and C. In this case, the composition of the relations propagates the two general constraints only and hence the result of the composition is the universal relation. The two reasoning rules proposed are also applicable in the temporal domain.

Consider the composition of the two relationships $overlap(e1, e2)$ and $overlap(e2, e3)$ shown in figure 4.13.

- The intersections of f_{02}, f_2, s_2 and p_{02} are of type I, described in section 2.3. Using rule 1, the following non-empty intersections are derived:

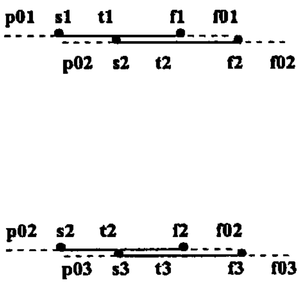
  $f_{01} \cap f_{03} \neq \phi \wedge f_{01} \cap f_3 \neq \phi \wedge f_{01} \cap t_3 \neq \phi \wedge t_1 \cap p_{03} \neq \phi \wedge s_1 \cap p_{03} \neq \phi \wedge p_{01} \cap p_{03} \neq \phi$.

  The intersection of t_2 yields indefinite intersections where $m' = 3 \wedge n' = 3$ and $\{f_{01}, f_1, t_1\} \sqsupseteq t_2 \sqsubseteq \{t_3, s_3, p_{03}\}$.

- Using rule 2 we get,

  $f_1 \cap f_{03} = \phi \wedge f_1 \cap f_3 = \phi$
  $t_1 \cap f_{03} = \phi \wedge t_1 \cap f_3 = \phi$
  $s_1 \cap f_{03} = \phi \wedge s_1 \cap f_3 = \phi \wedge s_1 \cap t_3 = \phi \wedge s_1 \cap s_3 = \phi$
  $p_{01} \cap f_{03} = \phi \wedge p_{01} \cap f_3 = \phi \wedge p_{01} \cap t_3 = \phi \wedge p_{01} \cap s_3 = \phi$

Combining the above constraints and selecting the strongest, the result matrix in figure 4.14 is derived. Using table 4.1 (mapping between component intersections and relations),



(a)

	f_{01}	f_1	t_1	s_1	p_{01}
f_{02}	1	0	0	0	0
f_2	1	0	0	0	0
t_2	1	1	1	0	0
s_2	0	0	1	0	0
p_{02}	0	0	1	1	1
	f_{02}	f_2	t_2	s_2	p_{02}
f_{03}	1	0	0	0	0
f_3	1	0	0	0	0
t_3	1	1	1	0	0
s_3	0	0	1	0	0
p_{03}	0	0	1	1	1

(b)

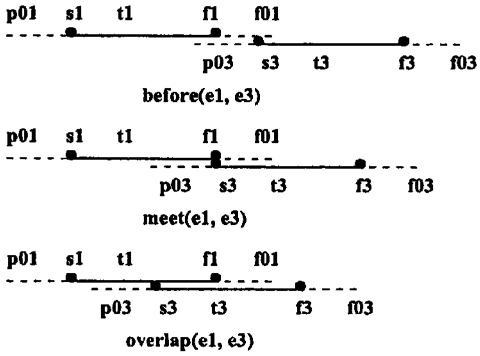
Figure 4.13: (a) An overlap relationship between two events. (b) adjacency matrix corresponding to the relationship in (a).

	f_{01}	f_1	t_1	s_1	p_{01}
f_{03}	*All*	$s_i, d_i, o_i, >$	$s_i, d_i, m_i, o_i, >$	$>$	$>$
f_3	$<, m, o, d, s$	$=, f, f_i$	s_i, d_i, o_i	m_i	$>$
t_3	$o, d, s, <, m$	o, d, s	$=, o, o_i, d, d_i, s, s_i, f, f_i$	d, f, o_i	$f, d, o_i, m_i, >$
s_3	$<$	m	o, f_i, d_i	$=, s, s_i$	$f, d, o_i, m_i, >$
p_{o3}	$<$	$<$	$<, m, o, f_i, d_i$	$<, m, o, f_i, d_i$	*All*

Table 4.1: Correspondence between the intersection of the components of temporal intervals as in figure 4.13 and temporal relations due to Allen. ($<$: before, $>$: after, m: meets, m_i: met by, o: overlap, o_i: overlapped by, s: starts, s_i: started by, d: during, d_i: contains, f:finishes, f_i: finished by, $=$:equal).

and using the steps described in the example of section 2.5, the three relations $<$, *meet* and *overlap* are propagated as shown in figure 4.14.

The above discussion handled simple intervals. The formalism can also be used to represent and reason over non-convex intervals such as that shown in figure 4.15, where P and R represent "pause" and "resume" of an activity.



	f_{01}	f_1	t_1	s_1	p_{01}
f_{03}	1	0	0	0	0
f_3	1	0	0	0	0
t_3	1	b_1	?	0	0
s_3	a_1	a_2, b_2	a_3	0	0
p_{03}	?	b_3	1	1	1

Figure 4.14: Result of the composition in figure 4.13 is a set of disjunctive relations $before(e_1, e_3) \vee meet(e_1, e_3) \vee overlap(e_1, e_3)$.

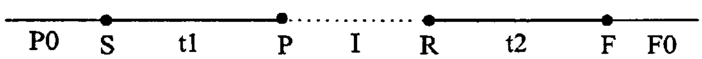


Figure 4.15: Decomposition of a simple non-convex interval with S = start, P = stop, R = restart, F= finish and I represent idle action.

4.4 Conclusions

In this chapter, the generality of the QSSR formalism has been demonstrated through its application on different types of spatial relations, namely, orientation and proximity relationships. A possible treatment of temporal events has also been proposed. The aim was to demonstrate the validity of the approach and how it can be applied in different contexts. Note, however, that the qualitative treatment can only complement, and not substitute, traditional quantitative techniques for manipulating space.

The following chapter looks into more depth in the combined treatment of space and time.

Chapter 5

Spatio-Temporal Representation and Reasoning

Time and space are primary dimensions in many application domains. Conceptually, time is an essential dimension for understanding and modeling space [CT95]. Modeling both space and time covers a wide spectrum of applications, including, medical and physical sciences and geographic and multimedia information systems. Several recent works to represent the genetic development of a human embryo [MNM+]are utilising methods of spatial and spatio-temporal representation. The work is still in early stages of development.

For example, by modeling spatio-temporal objects and relations in a genetic database, we could pose queries of the sort, "What is the effect of suppressing gene x on the growth of a group of cells in the first three weeks of embryonic development?".

Few works exist in the literature which address the problem of representing spatio-temporal relationships, or reasoning over spatio-temporal domains. The complexity of the spatial dimension compounded with change over time hindered the progress in this domain.

In this chapter, the QSRR approach proposed earlier is extended for the treatment of spatio-temporal domains. The approach is flexible and can be used with different levels of complexities in space and time. Spatio-temporal reasoning is treated essentially as

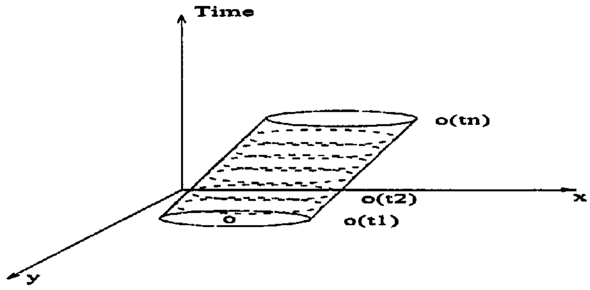


Figure 5.1: Dimensions of the Spatio-Temporal domain.

a constraint networks problem. Composition tables must be derived and used between objects of different types and dimensions. New spatio-temporal composition tables are derived. Considering the temporal dimension resulted in adding two new relationships in those table, i.e. an $n \times n$ composition table is extended to an $(n+2)^2$ table. For simplicity, simple spatial regions are used to demonstrate the ideas in this paper, but the method is readily extensible to handle different types and complexities of spatial objects.

5.1 The Problem Domain

A spatio-temporal problem domain extends the usual 2D (or 3D) Cartesian space by considering the time dimension. For simplicity, the presentation in this work is confined to 2D spatial objects. However, the approach is applicable and extensible to three dimensional objects. The dimensions of our domain are represented in figure 5.1.

A Spatio-Temporal Object

Spatial objects are considered to be functions of time. A spatial object O_i is defined as,

$$O_i = f(t)$$

Examples of spatio-temporal objects in GIS include, boundaries of vegetation regions, administrative or political boundaries which change over time. The behaviour of objects with respect to time is not necessarily uniform or strict. Other approaches in this domain usually consider temporal behaviour of objects through their start and end states only and ignore any intermediate states. Those approaches assume implicit linear and uniform behaviour over time.

In this work, this constraint is not imposed. Instead, every consistent behaviour of a spatial object is considered separately within the duration of time when this behaviour can be described by a single function. This spatio-temporal behaviour shall be denoted, *Episode* of an object. For example, the spatial properties of an object may change linearly over a certain period of time, followed by an interval of no change, then followed by an interval of non-linear (e.g. cyclic[1]) change. This history of an object can, therefore, be described using three different episodes.

An Episode

An episode is used to describe the behaviour of a spatial object over a certain period of time.

A quantitative definition of an episode e_i of an object o_j over the time interval t_1 and t_2 is as follows.

$$e_i(O_j) = \int_{t_1}^{t_2} f_i(t)$$

$f_i(t)$ is a function that holds between t_1 and t_2. I.e. an episode is a representation of all the states of the object between times t_1 and t_2. $e_i(O_j)$ has a value $(O_j)_{t_1}$ at time t_1, which is the spatial extent of the object at time t_1, i.e. $(O_j)_{t_1} = [f(t)]_{t_1}$ as shown in figure 5.2.

An episode is a coherent part of the history of an object. The full history of an object is a set of episodes.

$$history(O_j) = \sum_{i=1}^{n} e_i$$

[1]The change of coast-lines can be described by cyclic function while the phenomenon of continental drift can be described by a linear function.

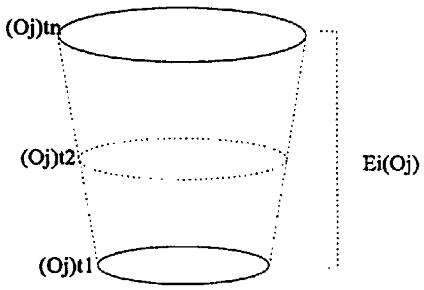


Figure 5.2: An episode defined as the combination of all object states in the time interval considered.

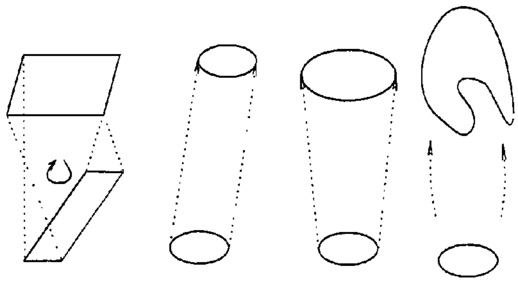


Figure 5.3: Some possible types of spatial change.

The function $f_i(t)$ can be used to qualitatively describe the different types of spatial changes that can an object may undergo.

Types of Change

Change can be classified according to the type and the rate of change. A spatial object can undergo any of four types of change in a spatio-temporal space, namely, translation (movement), rotation (change in direction), uniform or non-uniform scaling (change in size) or deformation (change in shape) or a combination of any of them. The different types of change are depicted in figure 5.3

The rate of change can be either 0 or $\neq 0$, corresponding to change or no change. A static episode is an episode through which the object does not undergo any change, i.e. its state

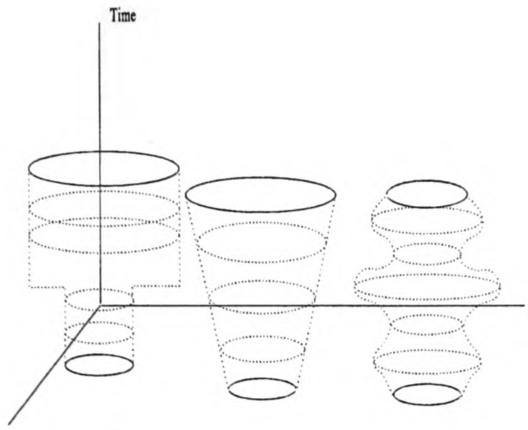


Figure 5.4: Some possible types of temporal change.

remains constant. Hence, a static episode is defined as,

$$\left[\frac{d(O_j)}{dt}\right]_{t_1}^{t_2} = 0$$

A dynamic episode is an episode through which the object has undergone one or more of the different types of change. It is defined as follows.

$$\left[\frac{d(O_j)}{dt}\right]_{t_1}^{t_2} \neq 0$$

Examples of types of spatio-temporal change are depicted in figure 5.4 in the case of scaling or change of size. An important type of change which does not involve spatial variation is the change of identity.

Spatio-temporal Relations

Spatio-temporal relations are studied between episodes of two objects and might be defined as functions of time.

$$R_k(e_i(O_j), e_m(O_k)) = f(t)$$

Two types of spatio-temporal relations can be identified:

Static Relations: A relation between two episodes is considered to be static if the relationship remains constant between the spatial objects involved during the whole

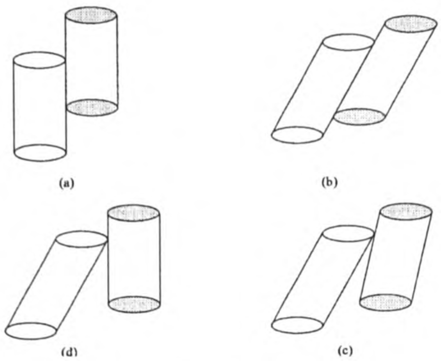


Figure 5.5: Examples of: (a) static episodes and static relations, (b) dynamic episodes and static relations, and (c) dynamic episodes and dynamic relations. (d) static,dynamic episode and dynamic relation.

interval of time considered. For example, if $R_1(e_i, e_m) = touch$ at any instant in time between t_1 and t_2, then R_1 is considered to be static in this time interval. A static relationship can be defined as follows.

$$\left[\frac{dR(e_i, e_m)}{dt}\right]_{t_1}^{t_2} = 0$$

Dynamic Relations: A spatio-temporal relation is considered to be dynamic if it is not static, i.e. the relation between the spatial objects changes during the interval of time considered. A dynamic relationship can be defined as follows.

$$\left[\frac{dR(e_i, e_m)}{dt}\right]_{t_1}^{t_2} \neq 0$$

Static relations may exist between either two static or two dynamic episodes. Dynamic relations exist only between two dynamic episodes or between a static and a dynamic episode. Examples of these relationships are shown in figure 5.5.

5.2 Representation of Episodes and Relationships between Episodes

A representation approach to the definition of spatio-temporal relations must satisfy the following requirements. It should be able to:

a. define the full set of temporal relations between episodes, namely, the set of 13 temporal relations between intervals and possible relations between intervals and points.

b. define the topological relations between the start states and end states of the two episodes considered.

c. define the topological relations between the start and end states of one episode and the body of the other episode in the case of non-equal temporal relationship between episodes.

d. define the set of topological relations between the bodies of the two episodes under consideration.

5.2.1 Representation of Episodes

Episodes can be generally represented by three components as shown in figure 5.6, namely, two spatial regions, representing the extent of the episode at the start and the end of the interval occupied by the episode (x_s and x_f), and a three dimensional spatio-temporal volume (x_i) representing the interior of the episode between t_s and t_f. This representation is also applicable in the case of the 4D spatio-temporal episodes.

An episode type can be fully described using four components that correspond to the four types of possible changes that it may undergo, as described in the previous section. Hence, an episode can be described by a tuple (L, R, C, D), where L is the translation component,

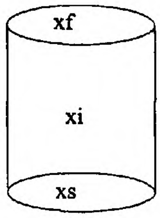


Figure 5.6: The three topological components of an episode: its start state x_s, its end state x_f and its interior (intermediate states) x_i.

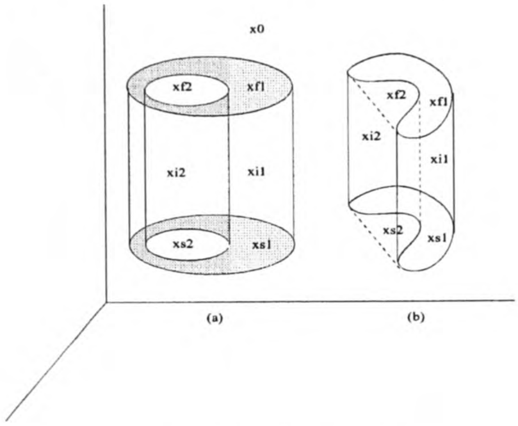


Figure 5.7: Examples of episodes for complex objects. (a) Region with a hole. (b) Concave region with a virtual cavity.

R, the rotation component, C, the scaling component and D, the deformation component. A value of 0 will be used to indicate no change and 1 to indicate change in that particular component. For example, $e_1(1, 0, 0, 0)$ represents a dynamic episode that has undergone a translational change. The value of the tuple is representative of the nature of change, i.e., $e(0, 0, 0, 0) = 0$ represents no change or a static episode.

The method of representation used here captures both the spatial and temporal characteristics without imposing any restrictions on the spatial type of the episode or its temporal characteristics. For example, the same decomposition strategy can be used for representing more complex objects, such as, regions with holes, or regions with virtual components, as illustrated in figure 5.7.

	x_0	x_{s1}	x_{s2}	x_{i1}	x_{i2}	x_{f1}	x_{f2}
x_0	-	1	1	1	0	1	1
x_{s1}	1	-	1	1	0	0	0
x_{s2}	1	1	-	0	1	0	0
x_{i1}	1	1	0	-	1	1	0
x_{i2}	0	0	1	1	-	0	1
x_{f1}	1	0	0	1	0	-	1
x_{f2}	1	0	0	0	1	1	-

(a)

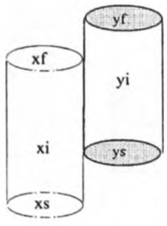
x_0						
1	x_{s1}					
1	1	x_{s2}				
1	1	0	x_{i1}			
0	0	1	1	x_{i2}		
1	0	0	1	0	x_{f1}	
1	0	0	0	1	1	x_{f2}

(b)

Figure 5.8: (a) Adjacency matrix of the shape in 5.7(a). (b) Half the symmetric adjacency matrix is sufficient to capture the representation of the episode.

An episode is assumed to be embedded in an infinite, connected spatio-temporal space. It is assumed to be connected and no overlap is possible between its constituting components. The topology of the episode and the embedding space can then be described by a matrix whose elements represent the connectivity relations between its components. This matrix shall be denoted *adjacency matrix.*

In figure 5.8(a), the representation of the episodes in figure 5.7(b) are presented. Each region is represented by two areal components x_s and x_f and infinite areal component x_0 representing the surrounding area. The fact that two components are connected is represented by a (1) in the adjacency matrix and by a (0) otherwise. Since connectivity is a symmetric relation, the resulting matrix will be symmetric around the diagonal. Hence, only half the matrix is sufficient for the representation of the object's topology and the matrix can be reduced to the structure in figure 5.8(b). In the decomposition strategy, the complement of the object in question shall be considered to be infinite. The suffix 0 (x_0) is used to represent this component. Note that x_0 represents the spatio-temporal embedding space.



(a)

	y_s	y_i	y_f
x_s	disjoint	disjoint	disjoint
x_i	touch	touch	disjoint
x_f	disjoint	touch	disjoint

(b)

Figure 5.9: (a) A simple relationship between two episodes. (b) Its corresponding component-relations matrix.

5.2.2 Representation of Spatio-Temporal Relations

Spatio-temporal relations between episodes can be represented by the combined spatio-temporal relations between their constituting components. Distinction of topological relations is dependent on the strategy used in the decomposition of the objects and their related spaces.

For example, in figure 5.9(a), a simple relationship between two episodes of simple regions x and y is shown. This relationship is uniquely represented by coding the individual relationships between the different components in a matrix structure, denoted, the *Component-Relations Matrix*, as shown in 5.9(b).

It will now be shown how the representation method developed satisfy the requirements identified in section 4 above.

a. If the start or/and end components of two episodes are connected or intersect, i.e. not disjoint, then it can be inferred that they co-exist temporally. It is sufficient, in this case, to represent their spatial relationships. If the components are disjoint, then it is necessary to distinguish their temporal relationships.

   Three possible types of disjoint relationships can be identified. These are illustrated in figure 5.10.

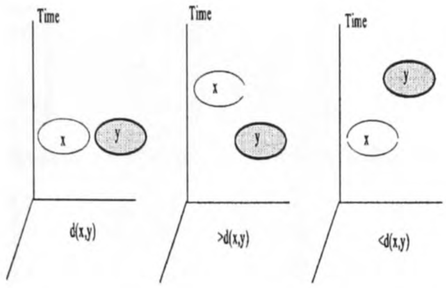


Figure 5.10: Different types of disjoint in the spatio-temporal domain.

(a) $d(x, y)$: indicates that object x is spatially disjoint from y and both existed at the same time point.

(b) $< d(x, y)$: indicating that object x is spatially disjoint from y and existed before y.

(c) $> d(x, y)$: indicating that object x is spatially disjoint from y and existed after y.

Hence, the set of all possible temporal relationships between episodes, as well as the topological relations between start and end states (requirements 1 and 2) can be represented by the set of 10 spatio-temporal relations between states and end-states. The two temporal disjoint relationships will exist, irrespective of the complexity of the objects considered. Hence, in the case of simple convex regions, the set of eight topological relations is expanded to ten relationships and so on. For other spatial objects the set is equal to the topological set + 2.

b. The third requirement studies the case of non-equal temporal relation between episodes. In this case there must exist a relation between the start or end state and the body of the other episode (between end states), i.e. a relationship between a 2D and a 3D object. This relation is equivalent to a 2D spatial relation between a 2D component of one episode (start and end states) and another 2D component of the other episode (cross-section of the body of an episode). If the objects considered are simple convex regions, then eight possible relationships can be distinguished. Different notations are used for naming the relations as shown in figure 5.11. Hence,

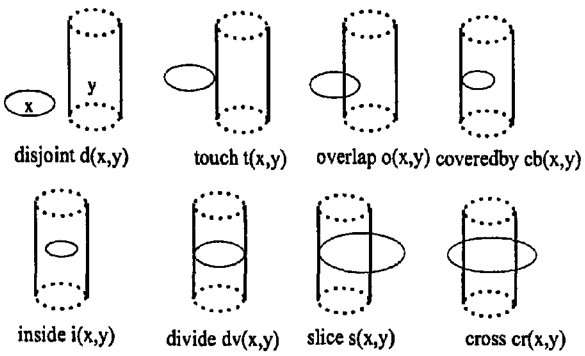


Figure 5.11: Different types of relationships between a region (representing an object state) and a volume (representing the interior of episodes).

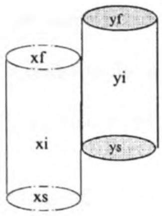
the domain of relations in this case is the set of eight topological relations between convex regions.

c. In the fourth requirement, the topological relations between the bodies of the episodes, i.e. between two volumes, are considered. The full set of relationships between two convex volumes in space consist of eight topological relations similar to those in the case of two simple regions in 2D space. Hence, the six overlap relationships in figure 5.11 are uniquely distinguished using the method.

From the above, the relations matrix in figure 5.9 can be modified as shown in figure 5.12.

Other examples of spatio-temporal relationships are shown in figure 5.13 along with their corresponding relations matrices.

Every instance of the component-relations matrix contains a collection of unique relationships between the components, and hence, overall uniqueness and soundness of representation are guaranteed. I.e. every possible relationship between two episodes will have a corresponding matrix which uniquely distinguishes it.

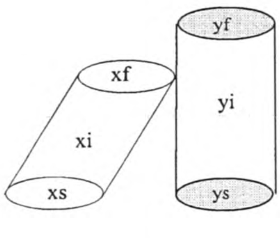


(a)

	y_s	y_i	y_f
x_s	$< d$	d	$< d$
x_i	t	t	d
x_f	$> d$	t	$< d$

(b)

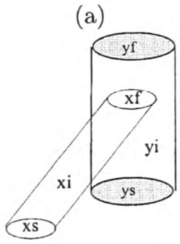
Figure 5.12: (a) A simple relationship between two episodes. (b) Its corresponding relation matrix, with different types of spatio-temporal disjointness.



(a)

	y_s	y_i	y_f
x_s	d	d	$< d$
x_i	d	d	d
x_f	$> d$	t	$< d$

(b)



(c)

	y_s	y_i	y_f
x_s	$< d$	d	$< d$
x_i	d	o	d
x_f	$> d$	cb	$< d$

(d)

Figure 5.13: Examples of relations between episodes and their relations matrices.

5.3 Spatio-Temporal Reasoning

Spatio-temporal reasoning is carried out between episodes in a constraint-network fashion. The composition of relations is achieved by combining the Relation matrices, and propagating the relations between every pair of components of the two episodes.

The pre-requisites for carrying out the reasoning mechanism are as follows.

a. Composition tables for objects, or end-states. In our case, composition tables between simple regions need to be extended to include the different types of spatio-temporal disjointness. The extended table is shown in table 5.1.

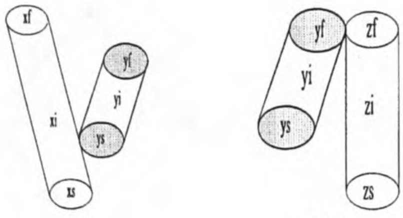
b. Composition tables for end-states and bodies of the episodes, i.e. between regions and volumes. Four different tables are required to handle the different combination. The tables could be computed using the spatial reasoning approach developed in chapter 2. Note that all the entries in the composition table for region-volume and volume-region are disjunctive sets of all ten spatio-temporal relations between regions and hence the table can propagate no new information. The composition table between volume-region and region-volume is shown in table 5.2 as one example of the four tables.

c. Composition tables between the bodies of the episodes, i.e. between two volumes in our case. Since we can reduce the relations between two volumes to the same set of relations between two simple regions, the composition table in this case will be similar to that produced for simple regions.

Some other examples of the reasoning process follows.

Example 1

Consider the relationships between the episodes of objects x, y and z as shown in figure 5.14(a) and (b). Their Relation matrices are shown in 5.14(c).



(a) (b)

x_f	x_i	x_s		z_s	z_i	z_f
$>d$	t	$<d$	y_s	$>d$	d	$<d$
d	d	d	y_i	d	d	d
$>d$	d	$<d$	y_f	$>d$	d	t

(c)

Figure 5.14: (a) Composing the relationships in (a) and (b). (c) Corresponding relation matrices.

Spatio-temporal reasoning is achieved using the following steps.

a. Propagate, systematically, the relationships between all pairs of components from the different episodes using the components of the common object.

   Hence in the above example, the relationship between x_s and z_s is first derived using their relationships with y_s, y_i and y_f. Then, relationships are derived between x_s and z_i, followed by x_s and z_f, and so on. As an example, the derivation of the relationship between x_s and z_s and x_i and z_i is given below.

   (a) $R(x_s, z_s)$:

   From the matrices, the constraint network between the two components is shown in figure 5.15(a). Using the regions composition table, we have,

$$
\begin{aligned}
< d(x_s, y_s) \wedge > d(y_s, z_s) &\rightarrow All(x_s, z_s) \\
d(x_s, y_i) \wedge > d(y_i, z_s) &\rightarrow All(x_s, z_s) \\
< d(x_s, y_f) \wedge > d(y_f, z_s) &\rightarrow All(x_s, z_s)
\end{aligned}
$$

   By intersecting the resulting sets, we have the conclusion: $All(x_s, z_s)$.

   (b) $R(x_i, z_i)$:

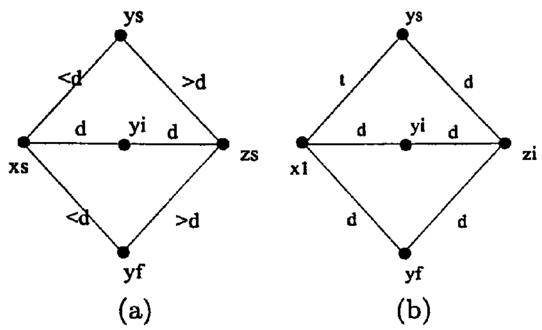


Figure 5.15: Constraint networks.

From the matrices, the constraint network between the two components is shown in figure 5.15(b). Using the regions composition table, we have,

$$\begin{aligned} t(x_i, y_s) \wedge d(y_s, z_i) &\rightarrow d(x_i, z_i) \vee t(x_i, z_i) \vee o(x_i, z_i) \vee ct(x_i, z_i) \vee cv(x_i, z_i) \\ d(x_i, y_i) \wedge d(y_i, z_i) &\rightarrow All(x_i, z_i) \\ d(x_i, y_f) \wedge d(y_f, z_i) &\rightarrow All(x_i, z_i) \end{aligned}$$

By intersecting the resulting sets, the composition yields a disjunctive set of relations: $d(x_i, z_i) \vee t(x_i, z_i) \vee o(x_i, z_i) \vee ct(x_i, z_i) \vee cv(x_i, z_i)$. This result can be more concisely represented by the negation of the complement set of relations, i.e. $\neg e(x_i, z_i) \wedge \neg ct(x_i, z_i) \wedge \neg cv(x_i, z_i)$.

b. The full set compositions are refined and presented in a matrix, denoted, the composition matrix, as follows.

	z_s	z_i	z_f
x_s	All	All	$< d$
x_i	All	$\neg i \wedge \neg cb \wedge \neg eq$	$\neg cv \wedge \neg ct \wedge \neg eq$
x_f	$> d$	d	$> d$

c. The constraints in the matrix are then combined and mapped to the resulting relation (or set of relations).

Identifying possible sets of relations

Soundness rules must be developed and used to map the composed relations matrix between the components of the episodes into a set of possible relations between the whole episodes. The development of such rules between objects of different types and dimensions is the subject of chapter 8.

For example, the composition of relations in the example above indicated that the possible relations between x_i and z_i may include $ct(x_i, z_i)$ or $cv(x_i, z_i)$. However, the resultant relations between x_i and z_f rules out those possibilities.

The case of static episodes is special, since the spatial relations between them are preserved with respect to time.

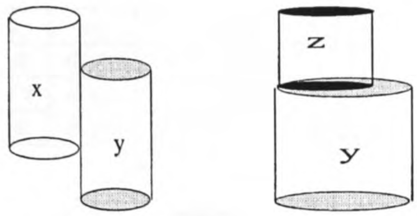
5.3.1 Spatio-temporal reasoning with Static Episodes and Relations

A static episode is an episode where spatial extents of the object remains constant in time. A static relation between episodes implies the episodes being involved in a static spatial relation during their coexistence. The episodes in figure 5.5(a) are static and are involved in a static relationship. In this case, a rule that govern the inter-relationships between the episode components can be stated as follow. If the interior of one episode $e(i)_1$ has a relationship R with the start $e(s)_2$ or end $e(f)_2$ of another episode e_2, then it must have the same relationship with the interior of that episode $e(i)_2$. I.e. $R(e(i)_1, e(s)_2) \vee R(e(i)_1, e(f)_2) \rightarrow R(e(i)_1, e(i)_2)$.

This rule can be incorporated in the reasoning process by deducing the relationship between x_i and z_i through the intersection of the set of relations between them and all the components of the other episode. I.e. $R(x_i, z_i) = R(x_i, z_s) \cap R(x_i, z_i) \cap R(x_i, z_f) \cap R(z_i, x_s) \cap R(z_i, x_f)$.

Example 2

Consider the relationships between the episodes of objects x, y and z as shown in figure



(a) (b)

x_f	x_i	x_s		z_s	z_i	z_f
$> d$	d	$> d$	y_s	$< d$	d	$< d$
d	t	t	y_i	d	d	d
$> d$	t	$< d$	y_f	cv	d	$< d$

(c)

Figure 5.16: (a) Composing the relationships in (a) and (b). (c) Corresponding relation matrices.

5.16(a) and (b). Their Relation matrices are shown in 5.16(c).

Spatio-temporal reasoning is achieved by propagating, systematically, the relationships between all pairs of components from the different episodes using the components of the common object.

The full set compositions are refined and presented in the composition result matrix, as follows.

	z_s	z_i	z_f
x_s	$< d$	$d \vee t \vee o \vee ct \vee cv$	$< d$
x_i	$d \vee t$	$d \vee t \vee o \vee ct \vee cv$	$d \vee t \vee o \vee ct \vee cv$
x_f	$> d$	All	All

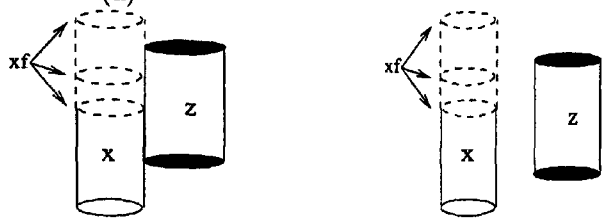
The relationship between x_i and z_i is calculated as follows.

$R(x_i, z_i) = \{d \vee t \vee o \vee ct \vee cv\} \cap \{d \vee t \vee o \vee ct \vee cv\} \cap \{All\} \cap \{t \vee d\} \cap \{d \vee t \vee o \vee cv \vee ct\} = \{d \vee t\}$.

It is noted that the matrix in not optimal, in the sense that some combinations may be impossible. Soundness rules need to be devised to exclude those cases. For static relations,

	z_s	z_i	z_f
x_s	$< d$	$d \vee t$	$< d$
x_i	$d \vee t$	$d \vee t$	$d \vee t$
x_f	$> d$	$d \vee t$	$d \vee t \vee > d \vee < d$

(a)



(b) (c)

Figure 5.17: (a) The resulting matrix from the composition of relations in figure example 2 . (b) The possible types of resulting relations.

it can be proved that if the initial propagation result is indefinite for the relations between the interiors of episodes and other elements, including bounding states, uncertainty must stem from the same disjunctive set of relations (with the addition of temporal disjointness). Hence, the same approach used for refining the relationship between x_i and z_i can be used for other indefinite relations. Hence, by intersecting $R(x_i, z_i)$ with other indefinite results the composition result matrix can be restated as shown in figure 5.17(a). The matrix can correspond to one of six possible relationships shown in 5.17(b) and (c).

5.4 Conclusions

An approach was presented for the representation and reasoning over spatio-temporal relationships. A spatio-temporal object is defined and then used to define the notion of an *Episode* which is a collection of object states in a specific temporal interval defined between a start and an end state. The topology of the episode is defined by decomposing it into components representing its start, interior and end. Spatio-temporal reasoning is carried out over episodes by composing the relationships between their comprising components in a constraint network fashion. The following issues are also presented.

a. Different types of episodes are identified, namely, static and dynamic, reflecting the nature of change of spatio-temporal objects over time.

b. Spatio-temporal relationships are classified into static and dynamic according to whether spatial relationships between objects remain unchanged in time.

c. Spatial composition tables are extended by adding three different types of temporal disjointness, namely, $< d$, $> d$, and d. When objects are not disjoint, they are considered to be co-existing in time.

d. Different composition tables for relationships between bounding states and interiors of episodes are derived using the method in chapter 2.

e. In the case of static episodes, soundness rules are defined and used to prune the compositions result into physically possible relationships.

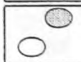
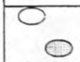
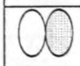
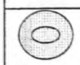
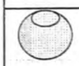
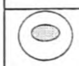
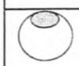
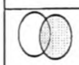
	$< d$ 	$> d$ 	d 	t 	i 	cb 	ct 	cv 	o 
	$< d$	all	$< d$	$< d$	$< d$	$< d$	$< d$	$< d$	$< d$
	all	$> d$	$> d$	$> d$	$> d$	$> d$	$> d$	$> d$	$> d$
	$< d$	$> d$	All	$d \vee t \vee i \vee cb \vee o$	$d \vee t \vee i \vee cb \vee o$	$d \vee t \vee i \vee cb \vee o$	d	d	$d \vee t \vee i \vee cb \vee o$
	$< d$	$> d$	$d \vee t \vee ct \vee cv \vee o$	$d \vee t \vee = \vee cb \vee cv \vee o$	$i \vee cb \vee o$	$t \vee i \vee cb \vee o$	d	$d \vee t$	$d \vee t \vee i \vee cb \vee o$
	$< d$	$> d$	d	d	i	i	All	$d \vee t \vee i \vee cb \vee o$	$d \vee t \vee i \vee cb \vee o$
	$< d$	$> d$	d	$d \vee t$	i	$i \vee cb$	$d \vee t \vee ct \vee cv \vee o$	$d \vee t \vee = \vee cb \vee cv \vee o$	$d \vee t \vee i \vee cb \vee o$
	$< d$	$> d$	$d \vee t \vee ct \vee cv \vee o$	$ict \vee cv \vee o$	$= \vee i \vee cb \vee ct \vee cv \vee o$	$ct \vee cv \vee o$	ct	ct	$ct \vee cv \vee o$
	$< d$	$> d$	$d \vee t \vee ct \vee cv \vee o$	$t \vee ct \vee cv \vee o$	$i \vee cb \vee o$	$= \vee cb \vee ct \vee o$	ct	$ct \vee cv$	$ct \vee cv \vee o$
	$< d$	$> d$	$d \vee t \vee ct \vee cv \vee o$	$d \vee t \vee ct \vee cv \vee o$	$i \vee cb \vee o$	$o \vee cb \vee i$	$d \vee t \vee ct \vee cv \vee o$	$d \vee t \vee ct \vee cv \vee o$	All

Table 5.1: The composition table between for spatio-temporal regions.

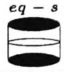
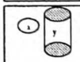
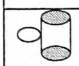
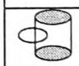
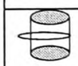
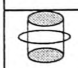
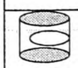
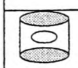
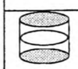
	d 	t 	o 	$cv-s$ 	$ct-s$ 	$cb-s$ 	$i-s$ 	$eq-s$ 
	*All*	$d \vee t \vee i \vee cb \vee o$	$d \vee t \vee o \vee cb \vee i$	$d \vee t \vee o$	$d \vee t \vee o$	$d \vee t \vee o \vee cb \vee i$	$d \vee t \vee o \vee cb \vee i$	$d \vee t \vee o$
	$d \vee t \vee o \vee cv \vee ct$	$d \vee t \vee o \vee = \vee cv \vee cb$	$d \vee t \vee o$ $i \vee cb$	$d \vee t \vee o$	$d \vee t \vee o$	$t \vee o \vee i$ cb	$o \vee cb \vee i$	$t \vee o$
	$d \vee t \vee o \vee cv \vee ct$	$d \vee t \vee o \vee cv \vee ct$	*All*	$d \vee t \vee o \vee ct \vee cv$	$d \vee t \vee o \vee ct \vee cv$	$o \vee cb \vee i$	$o \vee cb \vee i$	o
	$d \vee t \vee o$	$d \vee t \vee o$	$d \vee t \vee o \vee i \vee cb$	$d \vee t \vee o$ $= \vee cv \vee cb$	$d \vee t \vee o \vee cv \vee ct$	$o \vee cb \vee i$	$o \vee cb \vee i$	$o \vee cb$
	$d \vee t \vee o$	$d \vee t \vee o$	$d \vee t \vee o \vee i \vee cb$	$d \vee t \vee o \vee i \vee cb$	*All*	$o \vee cb \vee i$	$o \vee cb \vee i$	$o \vee i \vee cb$
	$d \vee t \vee o \vee ct \vee cv$	$t \vee o \vee cv \vee cv$	$o \vee cv \vee ct$	$o \vee cv \vee ct$	$o \vee cv \vee ct$	$o \vee = \vee ct \vee cv$	$o \vee cb \vee i$	$o \vee cv$
	$d \vee t \vee o \vee cv \vee ct$	$o \vee cv \vee ct$	$o \vee cv \vee ct$	$o \vee cv \vee ct$	$o \vee cv \vee ct$	$o \vee cv \vee ct$	$o \vee = \vee i \vee cb \vee cv \vee ct$	$o \vee ct \vee cv$
	$d \vee t \vee o$	$t \vee o$	o	$o \vee cv$	$o \vee ct \vee cv$	$o \vee cb$	$o \vee i \vee cb$	$o \vee = \vee cv \vee cb$

Table 5.2: The composition table between spatio-temporal regions (end-states of objects) and volumes (bodies of episodes).

Chapter 6

Reasoning without Composition Tables

Results of the spatial reasoning process usually documented in tables known as Composition tables. Examples of such tables have been presented in earlier chapters. Essentially, a composition table is a compilation of the combinatorial compositions of all sound spatial relationships between the objects involved. Hence, different composition tables need to be constructed for every different combination of objects and object types; a major challenge to QSRR as noted in [RCC92b]. Building such tables is useful and probably an essential step for the realisation of a qualitative reasoning engine in a spatial database. These tables have to be either stored initially or derived on the fly. Given the inordinate number of spatial objects, both options are not practically feasible for storage and computational costs.

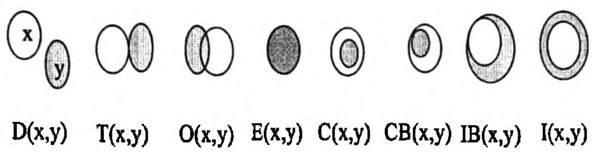
Another challenge for spatial reasoning is handling incomplete and uncertain knowledge in space. The ability to handle a certain level of indeterminacy makes techniques of QSRR attractive. Uncertainty with regards to spatial relationships is normally represented as a disjunction of the set of relations. The composition of such relations will involve more than one look up operation in the composition table as well as a summation operation. For example, if for convex regions A, B and C, we have the relations $touch(A, B) \vee$

overlap(A, B) and *inside*$(B, C) \vee$ *overlap*(B, C). The inference of the possible relations between objects A and C shall involve four look up operations in the composition table for each alternative combination as well as final summation operation. Using the composition tables in this way degrades the efficiency of the reasoning process [RCC92b].

In this chapter, a general algebra for space is presented that eliminates the need for composition tables and that also allows the application of spatial reasoning with incomplete or uncertain knowledge of topological relations.

6.1 Mapping Component Intersections into Relations

The intersection matrix is in fact a set of intersection constraints whose values identifies specific spatial relationships. Figure 6.1 represents the mapping between intersection constraints and set of spatial relations in the case of two convex areal objects. Table entries represent the resulting set of possible relations if the result of the intersection of the corresponding components is non-empty (or (1)). For example, if $x_0 \cap y_2 = 1$ is the only intersection known then the relationship between objects x and y is $D \vee T \vee O \vee I \vee IB$, and so on. If the result of the intersection is the empty set (or (0)), then the possible set of relations will be the complement of the sets shown. If $x_0 \cap y_2 = 0$, then the corresponding table entry will be $E \vee C \vee CB$. The possible relations between objects x and y can therefore be derived from the combination (set intersection) of all the table entries. An example is given in figure 6.2 where in 6.2(a) the intersection matrix for objects x and y is shown (with unknown value for $x_2 \cap y_2$). In 6.2(b) the mapping of the intersections into possible relations is given using the table (and its complement) in figure 6.1(b). The spatial relations between objects x and y is then derived from the set intersection of all table entries to be $I(x, y) \vee IB(x, y)$ shown in 6.2(c).



(a)

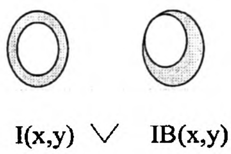
	y_0	y_1	y_2
x_0	ALL	$(D \vee T \vee O \vee I \vee IB)$	$(D \vee T \vee O \vee I \vee IB)$
x_1	$(D \vee T \vee O \vee C \vee CB)$	$(O \vee I \vee IB \vee C \vee CB \vee E)$	$(O \vee C \vee CB)$
x_2	$(D \vee T \vee O \vee C \vee CB)$	$(I \vee IB \vee O)$	$(T \vee O \vee IB \vee CB \vee E)$

(b)

Figure 6.1: (a) Set of possible relations between two convex areal objects. (b) Mapping the non-empty intersection of object components into a disjunctive set of possible relations.

	y_0	y_1	y_2
x_0	1	1	1
x_1	0	1	0
x_2	0	1	$0 \vee 1$

(a)



(c)

	y_0	y_1	y_2
x_0	ALL	$(D \vee T \vee O \vee I \vee IB)$	$(D \vee T \vee O \vee I \vee IB)$
x_1	$(I \vee IB \vee E)$	$(O \vee I \vee IB \vee C \vee CB \vee E)$	$(I \vee IB \vee O \vee T \vee E)$
x_2	$(I \vee IB \vee E)$	$(I \vee IB \vee O)$	$(T \vee O \vee IB \vee CB \vee E)$ OR $(D \vee I \vee C)$

(b)

Figure 6.2: (a) An example intersection matrix with unknown value for $x_2 \cap y_2$. (b) Mapping intersections of individual components to possible relations. (c) Possible relations between objects as the common subset in all the table entries.

6.1.1 The Space Algebra

The process of spatial reasoning can be defined as the process of propagating the intersection constraints of two spatial relations (for example, $R_1(A, B)$ and $R_2(B, C)$), to derive a new set of intersections between objects. The derived constraints can then be mapped to a specific spatial relation (i.e. the relation $R_3(A, C)$).

Let $X = \bigcup_{i=1}^{m} x_i$ and $Z = \bigcup_{k=1}^{n} z_k$ represent the spaces X and Z associated with objects x and z respectively. m and n are the total number of components in those spaces. If $y_j \subset Y$ and Y is the embedding space for the common object in the composition relation and since $X = Y = Z$, it follows that $(X \supset y_j) \wedge (y_j \subset Z)$.

In general the intersection of two components can take one of three values: 0 or 1 or ? where 0 indicates an empty intersection, 1 a non-empty intersection and ? indicates either a 0 or a 1. (unknown value). Hence, if $P = \{0, 1, ?\}$ then $y_j \subset X \rightarrow \forall x_i \in X(y_j \cap x_i = P_q)$ where $P_q \in P$. Similarly, $y_j \subset Z \rightarrow \forall z_k \in Z(y_j \cap z_k = P_q)$

The reasoning process can be carried in two steps, namely:

a. **Multiplication Operation:** on the intersection relations between every component from the intermediate space and every component of the other two spaces.

b. **Addition Operation:** on the results of multiplication for all the components of the intermediate space.

The multiplication operation can be expressed as follows:

$$(X \cap Z)_{y_j} = \left(\bigcup_{i=1}^{m} x_i \cap y_j\right) \star \left(\bigcup_{k=1}^{n} z_k \cap y_j\right) \tag{6.1}$$

where $(X \cap Z)_{y_j}$ are the propagated intersection relations between components of the spaces X and Y based on their intersection with the component y_j of space Y. The addition operation can be expressed as follows:

$$(X \cap Z) = \Sigma_{j=1}^{l}(X \cap Z)_{y_j} \tag{6.2}$$

where l is the total number of components of space Y.

Substituting 6.1 in 6.2 we get the general reasoning equation.

General Reasoning Equation

$$X \cap Z \;=\; \Sigma_{j=1}^{l}\left((\bigcup_{i=1}^{m} x_i \cap y_j) \;\star\; (\bigcup_{k=1}^{n} z_k \cap y_j) \right) \tag{6.3}$$

Note that there is no restriction on the application of equation 6.1 to a single or a set of components of the intermediate space, hence a general form for equation 6.1 can be stated as follows.

$$(X \cap Z)_{y'} = \left(\bigcup_{i=1}^{m} x_i \cap y' \right) \;\star\; \left(\bigcup_{k=1}^{n} z_k \cap y' \right) \tag{6.4}$$

where $y' \subseteq Y$, for example $y' = y_1 \cup y_2$..

The two equations 6.3 and 6.4 are the two general equation for spatial reasoning with incomplete or uncertain knowledge. Equation 6.3 shall always be applied in all cases. Equation 6.4 is only needed to be applied with equation 6.3 whenever a constraint exist of the following form:

$$x_i \cap y_j \;=\; ? \;\wedge\; x_i \cap y_{j+1} \;=\; ? \;\wedge\; x_i \cap (y_j \cup y_{j+1}) = 1 \tag{6.5}$$

i.e. the intersection of x_i with both y_j and y_{j+1} cannot be empty, then using $y' = (y_j \cup y_{j+1})$ will give $x_i \cap y' = 1$, i.e.,

$$((x_i \cap y_j) \;=\; 1 \;\vee\; (x_i \cap y_{j+1}) \;=\; 1) \tag{6.6}$$

To distinguish the constraint in 6.6 from a non related constraint of the form: $x_i \cap y_j \;=\; ? \;\wedge\; x_i \cap y_{j+1} \;=\; ? \;\wedge\; x_i \cap (y_j \cup y_{j+1}) =?$, a label will be used and equation 6.6 can be rewritten as: $x_i \cap y_j \;=\; ?_a \wedge x_i \cap y_{j+1} \;=\; ?_a$ The added letter a indicates that the two constraints are related.

A constraint of the type 6.6 can be either an input to the reasoning task or an output of it.

Conditions for Related Constraints ($?_\alpha$)

In the previous section it was shown how to deal with a related condition of the form $x_i \cap y_j =? \ \wedge \ x_i \cap y_{j+1} =? \ \wedge \ x_i \cap (y_j \cup y_{j+1}) = 1$ In this section the conditions under which such a constraint can result from the reasoning process are identified.

If x'' is the set of components of space X with all its components having a non-empty intersection with the component y_j of space Y. If x' is the set of components of space X with all its components having intersection values with component y_j of space Y as either 1 or ?. If m'' is the number of components of x'' and m' is the number of components of x', i.e. $x'' \subseteq x' \subset X$ and $m'' \subseteq m' \subseteq m$. Similarly, $z'' \subseteq z' \subset Z$ and $n'' \subseteq n' \subseteq n$.

Then the condition under which a related constraint will result from the reasoning process is as follows,

$$m'' > 0 \ \wedge \ n'' > 0 \ \wedge \ m' > 1 \ \wedge \ n' > 1 \tag{6.7}$$

For example, if $(x_1 \cap y_1 =? \wedge x_2 \cap y_1 = 1) \wedge (z_1 \cap y_1 = 1 \wedge z_2 \cap y_1 =?)$, where $m'' = n'' = 1$ and $m' = n' = 2$. then the result of the composition can be expressed as either:

- $x_1 \cap z_1 =? \wedge x_1 \cap z_2 =? \wedge x_2 \cap z_1 =? \wedge x_2 \cap z_2 =?$ with $x_2 \cap (z_1 \cup z_2) = 1 \wedge z_1 \cap (x_1 \cup x_2) = 1$, or,
- $(x_2 \cap z_1 =?_\alpha \wedge x_2 \cap z_2 =?_\alpha) \ \wedge \ (z_1 \cap x_1 =?_\beta \wedge z_1 \cap x_2 =?_\beta)$.

Note that if $m'' = m' = 1$, such that $x_1 \cap y_1 = 1 \wedge z_1 \cap y1 = 1 \wedge z_2 \cap y_1 =?$, then the result of the composition is $x_1 \cap z_1 = 1 \wedge x_1 \cap z_2 =?$.

Hence, there is a need to distinguish the relations where $m'' > 0 \wedge m' > 1$ and $n'' > 0 \wedge n' > 1$. We shall use the notations 1^+ and $?^+$ for values under the later condition.

$\star$	0	?	1	$?^+$	1^+
0	0	0	0	0	0
?	0	?	?	?	?
1	0	?	1	?	1
$?^+$	0	?	?	?	$?_\alpha$
1^+	0	?	1	?	$?_\alpha$

Table 6.1: Multiplication Table for incomplete knowledge.

$+$	0	?	1	$?_\beta$
0	0	?	1	$?_\beta$
?	?	?	1	$?_\beta$
1	1	1	1	1
$?_\alpha$	$?_\alpha$	$?_\alpha$	1	$?_\alpha \wedge ?_\beta$

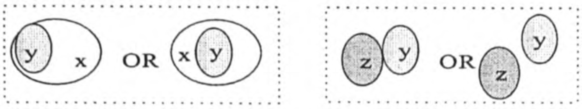
Table 6.2: Addition Table for incomplete knowledge.

Also note that the condition where $m = (m'' \vee m') \wedge n = (m'' \vee n')$ will result in a propagation of ? for all elements since it is an expression of the second general constraint.

Accordingly the multiplication and addition tables of our space algebra are as shown in figure 6.1 and 6.2.

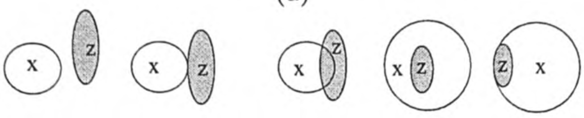
Note that in the addition table, $?_\alpha$ and $?_\beta$ were used since we add results for different components of space Y, i.e. $?_\alpha + ?_\beta = ?_\alpha \wedge ?_\beta$.

Equations 6.3 and 6.4 and the multiplication and addition tables represent the general space algebra for reasoning with incomplete or uncertain knowledge. The algebra makes no restriction on the complexity of the objects used or the completeness or uncertainty of the knowledge of the topological relations involved.



x_2	x_1	x_0		z_0	z_1	z_2
1	1	1	y_0	1	1	1
0	1	0	y_1	1	0	0
?	1	0	y_2	1	0	?

(a)



(b)

Figure 6.3: (a) Example reasoning problem with incomplete knowledge. (b) Resulting possible set of relations between x and z.

6.2 Example of Spatial Reasoning with Incomplete Knowledge

Consider the reasoning problem where the relations between simple convex areal objects x, y and z are: $C(x,y)$ *or* $CB(x,y)$ between x and y and $D(y,z)$ *or* $T(y,z)$ between y and z, as shown in figure 6.3(a). This indefiniteness is reflected in the intersection matrices in the figure where $x_2 \cap y_2 =?$ and $y_2 \cap z_2 =?$.

It is required to derive the possible relationships between objects x and z.

Applying the general topological reasoning equation 6.3 on y_0, y_1 and y_2 using the multiplication table, we get the following,

- y_0 intersections: ($n'' = n$ and $m'' = m$ - second general constraint)

$$(X \cap Z)_{y_0} \quad \rightarrow \quad \{x_0, x_1, x_2\} \cap \{z_0, z_1, z_2\} = 1$$

- y_1 intersections: $x_1 \cap z_0 = 1$ and $(X - x_1) \cap (Z - z_0) = \phi$.

- y_2 intersections: $(n'' > 0 \wedge n' > 1 \wedge m'' > 0 \wedge m' > 1$, i.e. a constraint of the type described in section 6.1.1(6.7) above.

  $x_1 \cap z_0 = ?_a$

  $x_1 \cap z_2 = ?_a$

  $z_0 \cap x_1 = ?_b$

  $z_0 \cap x_2 = ?_b$

  $x_1 \cap (z_0 \cup z_2) = 1$

  $x_2 \cap z_2 = ?$

  $z_0 \cap (x_1 \cup x_2) = 1$

Using the addition table we get the following:

	y_0	y_1	y_2	$\Sigma_{j=1}^{3} y_j$
$x_0 \cap z_0$	?	0	0	?
$x_0 \cap z_1$	?	0	0	?
$x_0 \cap z_2$	?	0	0	?
$x_1 \cap z_0$	?	1	$?_a \wedge ?_b$	1
$x_1 \cap z_1$	?	0	0	?
$x_1 \cap z_2$	?	0	$?_a$	$?_a$
$x_2 \cap z_0$	?	0	$?_b$	$?_b$
$x_2 \cap z_1$	?	0	0	?
$x_2 \cap z_2$	?	0	?	?

Also from the first general constraint $x_0 \cap z_0 = 1$. Compiling the above intersection we get the following resulting intersection matrix,

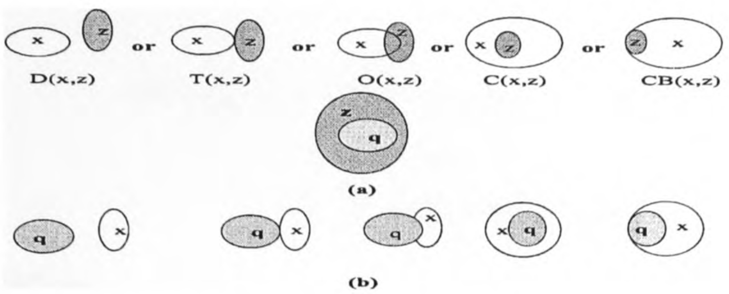


Figure 6.4: (a) Further composition of the spatial relations between x and z from the previous example with the relation $C(z, q)$ to get the possible relations between x and q in (b).

	z_0	z_1	z_2
x_0	1	?	?
x_1	1	?	?
x_2	?	?	?

Mapping the above matrix into spatial relations, as shown earlier in table 6.1(b), gives the disjunctive set: $R(X, Z) = D \vee T \vee O \vee C \vee CB$ as shown in figure 6.3(b).

6.2.1 Example 2

The resulting relationships in the above example gave the intersection of x_2 as $x_2 \cap \{z_0, z_1, z_2\} =?$, i.e. no constraints are propagated for the component x_2. If a new fact is added such that $x_2 \cap \{z_0, z_2\} = 1$, i.e. $x_2 \cap z_0 =?_a$ and $x_2 \cap z_2 =?_a$ and in this case the relationships between objects x and z are to be further composed with a relationship $C(z, q)$ between objects z and q as shown in figure 6.4.

Applying the general topological reasoning equation 6.3 on z_0, z_1 and z_2 and using the multiplication table, we get the following,

- z_0 intersections:

$$\begin{aligned}(X \cap Q)_{z_0} = \quad \rightarrow \quad & (x_0 \cap q_0 = 1) \wedge (x_1 \cap q_0 = 1) \\ \wedge \quad & (x_2 \cap q_0 = ?) \\ \wedge \quad & \{q_1, q_2\} \cap \{x_0, x_1, x_2\} = \phi\end{aligned}$$

- z_1 intersections:

$$\{q_0, q_1, q_2\} \cap \{x_0, x_1, x_2\} = ?$$

- z_2 intersections:

$$(q_0 \cap \{x_0, x_1, x_2\} = ?) \wedge \{q_1, q_2\} \cap \{x_0, x_1, x_2\} = \phi$$

From the previous example we have that $x_2 \cap z_0 = ?_a$ and $x_2 \cap z_2 = ?_a$.

Applying equation 6.4 for x_2 only, we get the following,

- x_2 intersections:

$$x_2 \cap (z_0 \cup z_2) = 1 \wedge (z_0 \cup z_1) \cap q_0 = 1 \rightarrow x_2 \cap q_0 = 1$$

Compiling the above intersection we get the following resulting intersection matrix,

	q_0	q_1	q_2
x_0	1	?	?
x_1	1	?	?
x_2	?	?	?

Mapping the above matrix into spatial relations, as shown earlier in table 6.1(b), gives the disjunctive set: $R(X, Q) = D \vee T \vee O \vee C \vee CB$ as shown in figure 6.4(b).

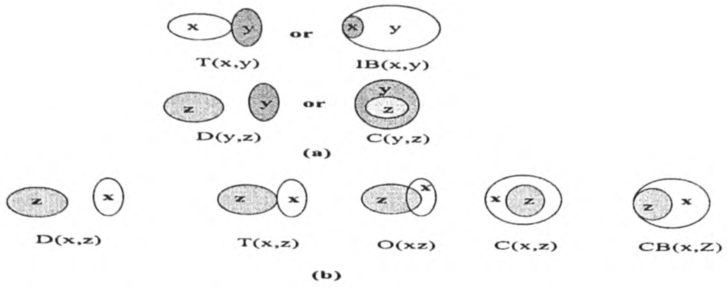


Figure 6.5: (a) Composition of relations which are non-conceptual neighbours, (b) the result of the composition between objects x and z.

6.2.2 Example 3: Composition with non-conceptual neighbours

In this example the space algebra is applied on incomplete knowledge which forms a disjunctive set of disjoint relations, i.e. non-conceptual neighbour relations, where conceptual neighbourhood is a phenomena defined by Freksa [Fre91b] in the temporal domain as "Two relations between pairs of events are conceptual neighbours if they can be directly transformed into one another by continuously deformation (i.e., shortening or lengthening) of the events topologically". In the spatial domain the same definition is valid by replacing "events" by "spatial objects".

Consider the relations between objects x and y and z as shown in figure 6.5 ($R(x,y) = T(x,y) \vee IB(x,y)$ and $R(y,z) = D(y,z) \vee C(y,z)$). Their representative intersection matrices are as follows:

x_2	x_1	x_0		z_0	z_1	z_2
1	1	1	y_0	1	?	?
?	?	?	y_1	1	?	?
1	?	?	y_2	1	0	0

Applying the general topological reasoning equation 6.3 on y_0, y_1 and y_2 and using the multiplication table, we get the following,

- y_0 intersections: ($n' = n$ and $m' = m$) - second general constraint

$$(X \cap Z)_{y_0} \quad \rightarrow \quad \{x_0, x_1, x_2\} \cap \{z_0, z_1, z_2\} = ?$$

i.e. no constraints are propagated.

- y_1 intersections: ($m' = m$ and $n' = n$) since the intersection of all the components of X with y_1 are all ? ($X \cap y_1 =?$, equation 6.3 will propagate no new constraints, i.e.

$$(X \cap Z)_{y_1} \quad \rightarrow \quad \{x_0, x_1, x_2\} \cap \{z_0, z_1, z_2\} = ?$$

- y_2 intersections: ($m'' = m' = 1$ and $n'' > 0$)
then $x_2 \cap z_0 = 1$.

Also from the first general constraint $x_0 \cap z_0 = 1$. Compiling the above intersection we get the following resulting intersection matrix,

	z_0	z_1	z_2
x_0	1	?	?
x_1	?	?	?
x_2	1	?	?

Mapping the above matrix into spatial relations, as shown earlier in table 6.1(b) gives the disjunctive set: $R(X,Z) = D \vee T \vee O \vee C \vee CB$ as shown in figure 6.5(b).

While in the case of a simple convex object the use of composition table seems simple (8 relations with 64 entries in the table), a slightly more complex object such as a convex region with indeterminate boundaries produces 44 relations giving rise to a table with 1936 entries (a table never developed). Hence, using composition tables for objects with random complexity is practically not feasible. The method presented above is a practical alternative to the use of composition tables.

6.3 Conclusions

In this chapter an approach is proposed for reasoning with incomplete topological knowledge in space, with respect to spatial relationships, without the need for composition tables. The approach builds on and generalizes the formalism presented in chapter 2 where spatial relations are represented by the intersection of object and space components. The reasoning method is applicable to objects of arbitrary complexity. One general equation is proposed here for the propagation of intersections between object components and the derivation of the result of spatial composition. A major advantage of this method is that reasoning with incomplete knowledge can be done by direct application of the equation and the algebra on different types of spatial objects, and thus eliminates the need for utilizing the inordinate number of composition tables which must be built for specific object types and topology. The method is applied on spatial objects of arbitrary complexity and in a finite definite number of steps controlled by the complexity needed in the representation of objects and the granularity of the spatial relations required.

Uncertainty with respect to object position and extension is considered in the next chapter.

Chapter 7

QSRR with Uncertainty

Precise information required in quantitative methods are sometimes neither available nor needed. For example, approximate expression of place names and locations is needed in general purpose geographic information systems and search engines. Representation of uncertain spaces is recognised as an important research topic with many application areas, for example, in the design of ontologies for geographic information retrieval over the web.

Recently, there has been an upsurge on explicit representation of imprecise and indeterminate regions [CD97]. Current approaches to representation and reasoning in space are mostly limited to handling simple objects in topological spaces. Proposals are generally extensions of existing approaches for representation in definite spaces and therefore carry their limitations.

In this chapter, a study of the notion of uncertainty is presented. Possible types of uncertainty are identified and the concept of the degree of uncertainty is clarified. Three modes of spatial uncertainty are also distinguished which relate directly to how the concept is represented and applied. A uniform model of representation in uncertain spaces is then presented and examples are used to demonstrate its validity for random object types and shapes. A reasoning formalism proposed earlier for propagation of spatial relations in definite spaces is extended for reasoning in uncertain spaces.

7.1 Types of Uncertainty

Different spatial attributes can be associated with an object in space to define, for example, its position, shape, configuration, orientation, etc. The accuracy of the representation of the object is directly dependent on the values of those attributes. To precisely define a spatial object, each of its associated properties must hold a unique value. However, this value may be one of a number of possible and correct values that can be associated with a spatial property. For example, Eiffel Tower as a place could be defined to be in Europe, in France, in Paris, or can be described exactly by its (x,y) map grid reference.

Hence, spatial uncertainty of objects in space occurs when one or more spatial attribute associated with an object holds more than one of a set of possible values. Example of values of spatial properties include, the points making its boundary, the set of component objects it is built from, or a description of the spatial arrangements of the object's parts. Also, uncertainty can be *partial*, associated with only a spatial property defining an object, such as its boundary.

Different types of spatial uncertainty can be defined as follows.

Positional uncertainty: where the precise location of an object or one of its components is not certain.

Extension uncertainty: where the spatial extent of the object's boundary or the boundary of one of its components is not certain, as shown in figure 7.1(a). The shaded ring in the figure represents the area within which the boundary may be found.

Configuration uncertainty: where the specific components making up a composite spatial object and their number are not certain. Figure 7.1(b) shows a region with holes where it is not known whether the component holes are A and B_1 or A and B_2.

Orientation uncertainty: where the orientation of the object, or the orientation of one of its components is not certain as shown in figure 7.1(c).

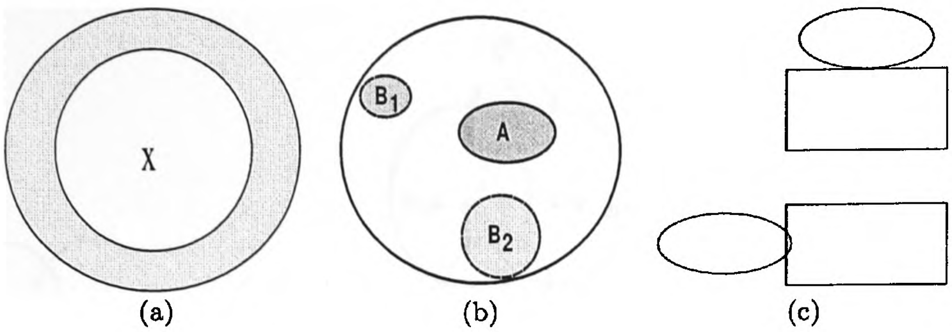


Figure 7.1: (a) Object extension uncertainty. (b) Configuration uncertainty. (c) Object orientation uncertainty

7.2 Modes of Uncertainty

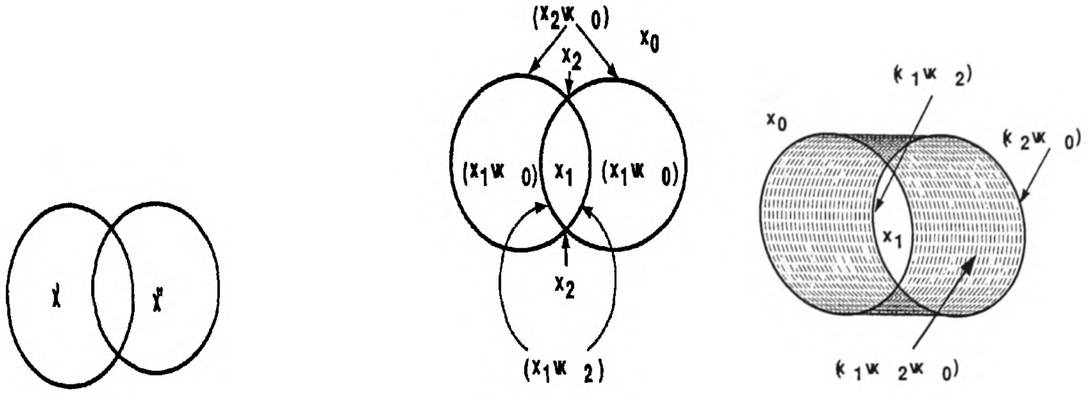
To illustrate the different modes of uncertainty, an example from the temporal domain is used, for the sake of simplicity of its one dimensional nature. Two modes of uncertainty can be distinguished, namely, discrete and range. An example of discrete uncertainty is expressing arrival time by the fact: "I will arrive at either 10 am or 11 am". An example of range mode of uncertainty, is when the arrival time is defined by a range of ordered values, for example, "I will arrive between 10:00 am and 11:00 am".

7.3 Representation of Uncertainty of object Properties

In this section a representation scheme for the different types and modes of spatial uncertainty is presented. The method is based on and extends the basic QSRR approach proposed in chapters 2 and 3. In what follows, examples of representation of objects with spatial uncertainty are given.

Representation of Object Location uncertainty:

Discrete location uncertainty can be represented by placing a copy of the object in each



(a) (b) (c)

Figure 7.2: (a) and (b) Discrete location uncertainty. (c) Range uncertainty.

of the possible values of the uncertain location, as shown in figure 7.2(a). In the figure, object x is represented by both copies $x' andx"$. Using the representation scheme as above, the space containing both copies is represented by the intersection of their respective components. The intersection of x' and $x"$ is a definite part of the component x_1. The rest of both $x' andx"$ can be either x_1 or x_0, and hence will labeled $(x_1 \vee x_0)$. Similarly, different parts of objects' boundaries can be labeled according to whether their comprising points are possibly part of either x_1 or x_2 or x_0 as shown in figure 7.2(b).

If the locations of x' and $x"$ represents the bounds of a range uncertainty as shown in figure 7.2(c), the representation changes to include the boundary as one of the possible components inside the two circles and between the circles and their tangents as shown. Also, the two points representing certain points on the boundary x_2 do not exist any longer.

Representation of object extension uncertainty:

A decomposition scheme in the case of range uncertainty is shown in figure 7.2(b) where the boundary x_2 of x can exist anywhere between $(x_1 \vee x_2)$ and $(x_2 \vee x_0)$. It is interesting

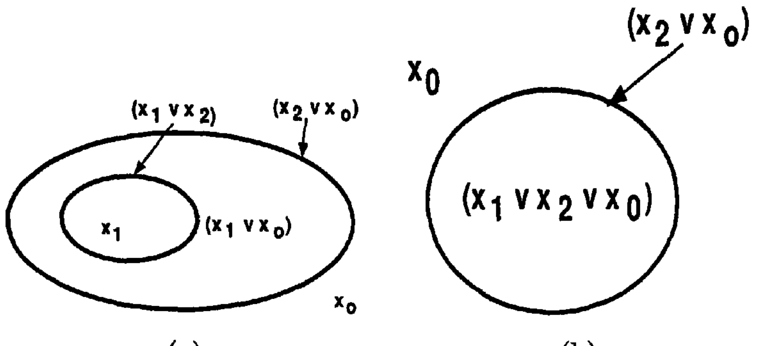


Figure 7.3: (a) Extent-Discrete uncertainty. (b) Combined location and extension uncertainty

to note that all related work on uncertainty of space was concerned mainly with range uncertainty over simple convex regions.

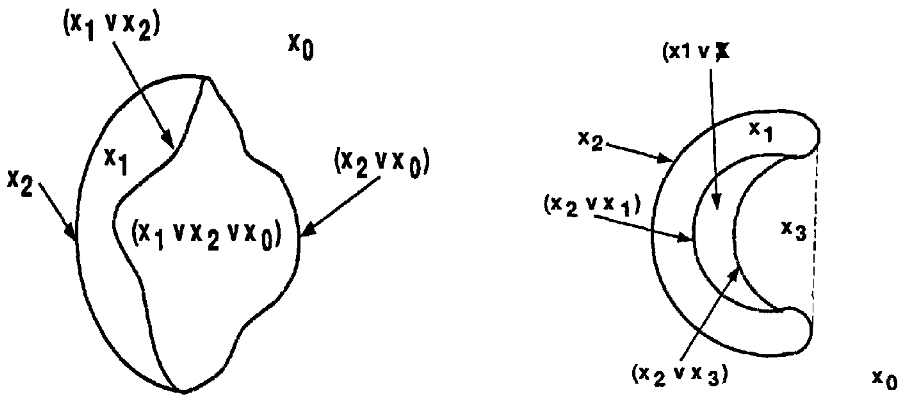
An example of discrete uncertainty of object's extension is shown in figure 7.3(a) where the boundary can exist either at $(x_1 \vee x_2)$ or $(x_2 \vee x_0)$. The area in-between must be either x_1 or x_2.

In the case of a combined location and extension uncertainty the representation in figure 7.3(b) is used where there are no assigned regions for x_1 only.

Partial uncertainty in the case of convex region is represented as in figure 7.4(a) where part of the boundary is definite (x_2) and the rest is bounded between $(x_1 \vee x_2)$ and $(x_2 \vee x_0)$. A similar partial range uncertainty in case of concave object is shown in figure 7.4(b) where the mode is discrete partial uncertainty since it's only the boundary of concavity which is under uncertainty.

Orientation and Spatial Arrangement Uncertainty

Orientation uncertainty is represented in a similar way to topological uncertainty above,



(a) (b)

Figure 7.4: (a) Partial Extent uncertainty. (b) Partial Extent-Range uncertainty.

e.g. (front $\vee$ right) as shown in figure 7.5(a). In figure 7.5(b), spatial arrangement uncertainty is shown, the object is a region with or without a hole, where the hole is expressed as $(x_3 \bigcup x_4)$.

As shown the method of representation of uncertainty is simple and general and will show in the following section how such scheme of representation lends itself to representation of relations and reasoning about them.

7.4 Representation of Spatial Relations in Uncertain Spaces

In this section, the representation of the topological relations through the intersection of their components is adopted and generalized for objects with spatial uncertainty. The complete set of spatial relationships are identified by combinatorial intersection of the components of one space with those of the other space.

Example: Range Uncertainty Relations

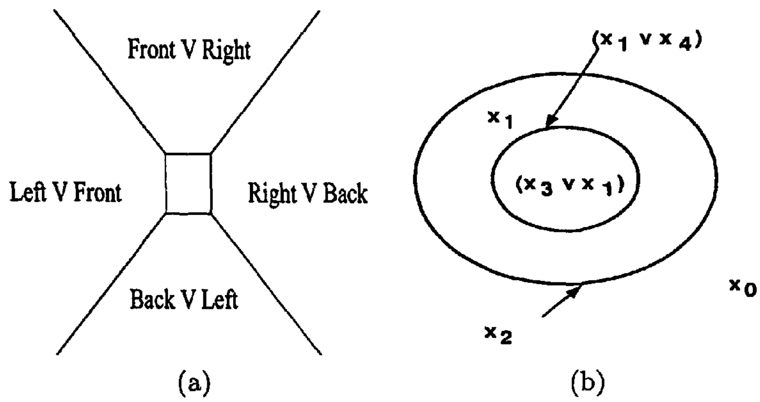
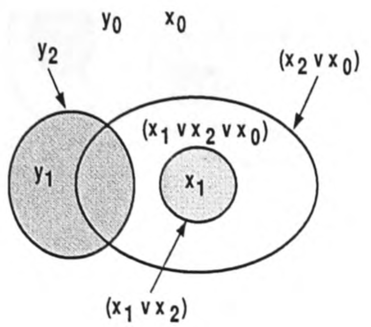


Figure 7.5: (a) Orientation uncertainty. (b)Spatial arrangement uncertainty.

Consider the relationship between objects x and y in figure 7.6(a). Object x is spatially uncertain, with range uncertainty mode. Object y is crisp. The intersection matrix representing the relationship is shown in 7.6(b). The intersection matrix can be rewritten by mapping the components uncertainty into intersections uncertainty between crisp objects in figure 7.7(a). On comparing the matrix with the set of of 8 relations between two simple crisp regions, a different representation of the relation in 7.6(a) can be described as a disjunctive set of relations $\{disjoint \vee touch \vee overlap\}$ as shown in figure 7.7(b).

Example: Discrete Extent Uncertainty Relations

Consider the relationship between objects x and y, both are vague in discrete mode. Both objects can have one of two configurations as shown in figure 7.8(a). The intersection matrix for this relation is in figure 7.8(b). Mapping this relation into corresponding set of crisp relations gives the relationships $disjoint \vee overlap \vee contain$.



(a)

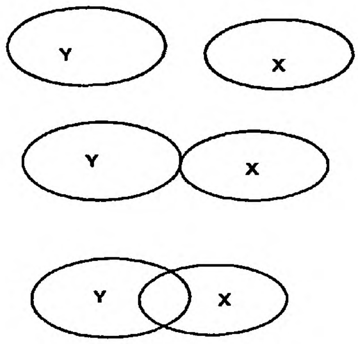
	y_1	y_2	y_0
x_1	0	0	1
$(x_1 \vee x_2)$	0	0	1
$(x_1 \vee x_2 \vee x_0)$	1	1	1
$(x_2 \vee x_0)$	1	1	1
x_0	1	1	1

(b)

Figure 7.6: (a) Relationship between a range-uncertain object x and a crisp object y (b) Corresponding intersection matrices.

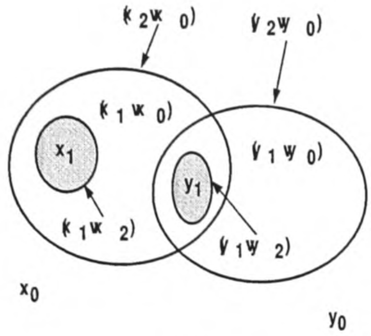
	y_1	y_2	y_0
x_1	$0 \vee 1$	$0 \vee 1$	1
x_2	$0 \vee 1$	$0 \vee 1$	1
x_0	1	1	1

(a)



(b)

Figure 7.7: (a) Mapped intersection relation. (b) Possible relations.



(a)

	y_1	$y_1 \vee y_2$	$y_1 \vee y_0$	$y_2 \vee y_0$	y_0
x_1	0	0	0	0	1
$x_1 \vee x_2$	0	0	0	0	1
$x_1 \vee x_0$	1	1	1	1	1
$x_2 \vee x_0$	0	0	1	1	1
x_0	0	0	1	1	1

(b)

Figure 7.8: Discrete extent uncertainty.

7.5 Reasoning with Uncertainty

7.5.1 Example: Reasoning with Range Extent Uncertainty

In [CDF01b], Clementini et al defined a set of 44 possible relations between objects with undetermined boundaries (range extension uncertainty). In this example, we use two of those relations, shown in figure 7.10, to demonstrate the composition of spatial relationships in uncertain spaces. In [CDF01b] objects were represented using three components, of boundary, interior and exterior. Using the proposed representation methodology above, objects are represented by the following three component: $\{x_1, (x_1 \vee x_0), x_0\}$, where the broad boundary is represented by the disjunctive set of possible components $(x_1 \vee x_0)$ and components $(x_1 \vee x_2)$ and $(x_2 \vee x_0)$ are omitted as shown in figure 7.9. However, to comply with the notations in [CDF01a] $(x_1 \vee x_0)$ will be called x_2.

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- y_1 intersections:

$$\{x_1, x_2\} \sqsupseteq y_1 \quad \wedge \quad y_1 \sqsubseteq \{z_1\}$$

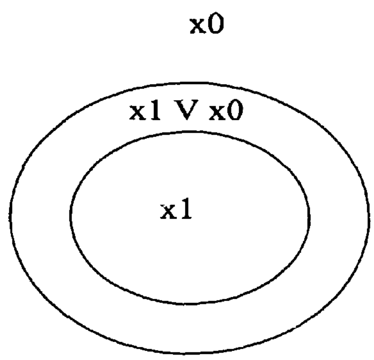
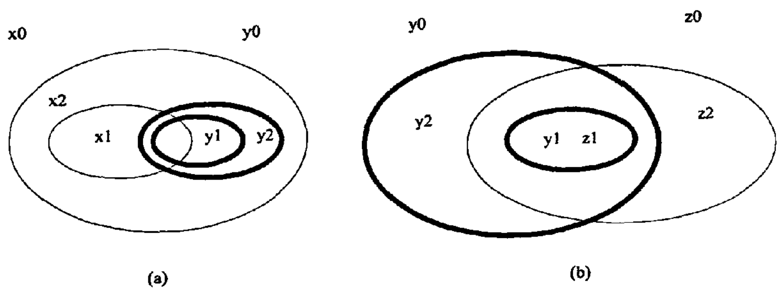


Figure 7.9: Broad boundary.



	y_1	y_2	y_0
x_1	1	1	1
x_2	1	1	1
x_0	0	0	1

(c)

	z_1	z_2	z_0
y_1	1	0	0
y_2	0	1	1
y_0	0	1	1

(d)

Figure 7.10: (a) and (b) Spatial relationships between vague regions x, y and z. (c) and (d) Corresponding intersection matrices.

$$\rightarrow \quad x_1 \cap z_1 \neq \phi \wedge x_2 \cap z_1 \neq \phi$$

- y_2 intersections:

$$\begin{aligned} \{x_1, x_2\} \sqsupseteq y_2 \quad & \wedge \quad y_2 \sqsubseteq \{z_2, z_0\} \\ & \rightarrow \quad x_1 \cap (z_2 \cup z_0) \neq \phi \wedge x_2 \cap (z_2 \cup z_0) \neq \phi \end{aligned}$$

- y_0 intersections:

$$\begin{aligned} \{x_1, x_2, x_0\} \sqsupseteq y_0 \quad & \wedge \quad y_0 \sqsubseteq \{z_2, z_0\} \\ & \rightarrow \quad x_1 \cap (z_2 \cup z_0) \neq \phi \wedge x_2 \cap (z_2 \cup z_0) \neq \phi \\ & \wedge \quad x_0 \cap (z_2 \cup z_0) \neq \phi \end{aligned}$$

Refining the above constraints, we get the following intersection matrix.

	z_1	z_2	z_0
x_1	1	a_1,c_1	a_2,d_1
x_2	1	b_1,c_2	b_2,d_2
x_0	0	?	1

Where a_1 and a_2 represent the constraint $x_1 \bigcap (z_2 \vee z_0) = 1$ and b_1 and b_2 represent the constraint $x_2 \bigcap (z_2 \vee z_0) = 1$, c_1 and c_2 represent the constraint $z_2 \bigcap (x_1 \vee x_2) = 1$ and d_1 and d_2 represent the constraint $z_0 \bigcap (x_1 \vee x_2) = 1$ and the ? represents $(1 \vee 0)$. The result matrix corresponds to one of four possible relationships between x and z, namely numbers 21, 22, 23 and 25, as shown in figure 7.12.

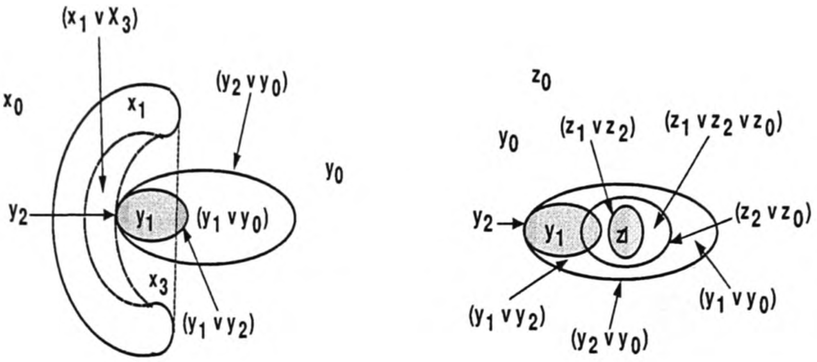
7.5.2 Example 2: Reasoning with Different Object Types in Uncertain Space

This example demonstrates reasoning with different objects, namely, a solid concave object x, (no distinction between boundary and interior of x), with discrete partial uncertainty, a convex object y with partial discrete uncertainty and convex object z with range uncertainty. The relations between the objects are shown in figure 7.11(a) and (b), and the corresponding intersection matrices are as shown in figure 7.11(c) and (d) respectively.

The composed relation matrix is as shown in figure 7.12(a), and the corresponding five possible configurations are in 7.12(b).

7.6 Conclusions

In this chapter, uncertainty in space is studied. Four types of spatial uncertainty were identified, related to the different spatial properties of objects, namely, positional, extension, configuration and orientation. The concept of the "modes" of uncertainty was also introduced. Spatial uncertainty operates in three different modes, namely, discrete, range and aggregate. Related approached have addressed the range uncertainty mode and were generally limited to handling simple object types, such as convex regions. To our knowledge, no work has addressed the problem of reasoning with uncertainty. In this chapter an exact approach, as opposed to fuzzy approaches, to the representation of uncertain space is proposed. Here, no gradual change between labels A and B is used, but the area of gradual change is considered to be either A or B. The model is flexible and handles the different types and modes of uncertainty homogeneously. The approach can also be used is situations of partial uncertainty of objects and relations. The representation method is complemented with a general reasoning formalism to propagate different types of relations in uncertain spaces.



(a) (b)

	x_1	$x_1 \vee x_3$	x_3	x_0
y_1	0	0	1	1
y_2	0	1	1	0
$y_1 \vee y_2$	0	0	1	1
$y_1 \vee y_0$	0	0	1	1
$y_2 \vee y_0$	0	0	1	1
y_0	1	1	1	1

(c)

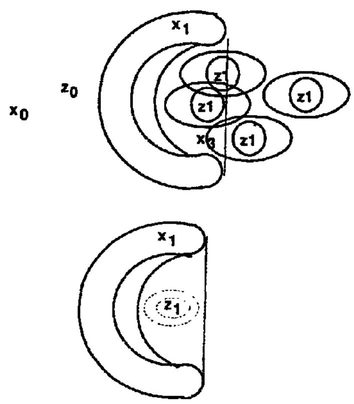
	z_1	$z_1 \vee z_2$	$z_1 \vee z_2 \vee z_0$	$z_2 \vee z_0$	z_0
y_1	0	0	1	1	1
y_2	0	0	0	0	1
$y_1 \vee y_2$	0	0	1	1	1
$y_1 \vee y_0$	1	1	1	1	1
$y_2 \vee y_0$	0	0	0	0	1
y_0	0	0	0	0	1

(d)

Figure 7.11: Object x with discrete-partial uncertainty, object y with partial discrete uncertainty and z with range uncertainty. Example relations and corresponding intersection matrices.

	z_1	$z_1 \vee z_2$	$z_1 \vee z_2 \vee z_0$	$z_2 \vee z_0$	z_0
x_1	0	0	0	0	1
$x_1 \vee x_3$	0	0	0	0	1
x_3	a	b	c	d	1
x_0	a	a	e	a	1

(a)



(b)

Figure 7.12: (a) Resulting composition matrix for figure 7.11. (b) Corresponding spatial relations.

Chapter 8

On Completeness and Soundness

Qualitative spatial representation and reasoning (QSRR) is an application of the general field of qualitative modeling, where the main goal is to model the state and behaviour of a given system. In a qualitative space, the states to be modeled are the spatial objects and their inter-relationships, and the behaviour is either static or dynamic.

In the case of static behaviour, a space containing a set of three or more spatial objects is studied to identify the possible or feasible set of relationships between those objects. The basic problem in this case is the composition of spatial relations stated as follows: given a relation R_1 between objects A and B and a relation R_2 between objects B and C, find the corresponding set of possible relations between objects A and C. For example, if $contains(A, B)$ and $overlaps(B, C)$, it should be concluded that the relations between A and C is either $contain(A, C)$ or $overlap(A, C)$.

In the case of dynamic behaviour, the model needs to compute the possible sequence of transitions between different states (or spatial relations). For example, figure 8.1 is a model of the sequence of transition of relations between two simple regions.

A basic requirement for a QSRR model to be complete and sound is to represent all the possible states (i.e. spatial relations) and to exclude any non-feasible ones. In this paper, this problem is addressed with spatial objects of arbitrary complexity and dimension. The

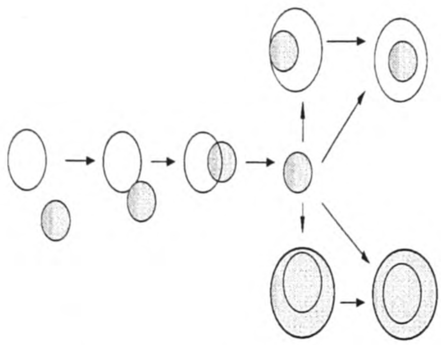


Figure 8.1: The possible states of transition between the various relations in the case of two simple regions.

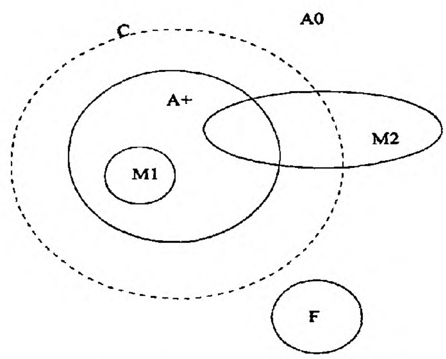


Figure 8.2: Different sets of spatial relations for characterising representation models.

approach is also valid in any space dimension.

Let A^+ be the set of complete sound relations between a pair of spatial objects, as shown in figure 8.2. A^+ is a finite set. If A_0 represents the (infinite) set of all invalid or non-sound states that can be modeled, then $A = A^+ \cup A_0$ is the set of all states that can be modeled.

The set A^+ is the complete (all) and sound (physically possible) set of relations between the two objects. Different categories of representation model can be identified.

a. An invalid representation model is a model which represents only invalid or non-sound relations. In figure 8.2, the set F is the set of invalid relations, where $F \not\subset A^+$ and $F \subset A_0$.

b. A sound, but incomplete model represents relations set M_1 in figure 8.2, where $M_1 \subset A^+$, $M_1 \not\subset A_0$ and $M_1 \neq A^+$.

c. A partially sound model represents relations set M_2 in figure 8.2, where $M_2 \cap A^+ \neq \phi$, $M_2 \cap A_0 \neq \phi$, $M_2 \not\supseteq A^+$ and $M_2 \not\subseteq A^+$.

d. A complete but not sound model represents relations set C in figure 8.2, where $A^+ \subset C$ and $C \cap A_0 \neq \phi$.

e. A complete and sound model represents relations set I, where $I = A^+$ and $I \cap A_0 = \phi$.

A QSRR formalism strives to be complete and sound. The above taxonomy is used in the following section to characterise the various approaches proposed in the literature

Two main approaches can be classified for modeling spatial relations. One starts by identifying the set of sound relations in the domain studied and then model the relations using constraints that define the different distinct states. The other approach starts by identifying the set of constraints that govern the space studied and use them to define the states or relations using those constraints. We denote the first approach a *relation-driven* approach and the second approach a *constraint-driven* approach. The first approach is sound, but with no guarantee of completeness and the second approach aims to achieve completeness but does not guarantee soundness.

This chapter is primarily concerned with topological relations. The spatial domain is rich with various possible types, dimensions and shapes of spatial objects. A vast number of possible relations may exist between those objects. It is therefore practically impossible for the relation-driven approaches to claim completeness. The model in this case can be represented by the set M_1 in figure 8.2, where M_1 is much smaller than A^+.

A primary method in this category is due to Randell et al [RCC92a]. Eight relations between simple convex regions are axiomatised using the concept of connection. Attempts have been made to extend the formalism by introducing different taxonomies of relationships between concave regions [Coh95] and regions with in-determined boundaries [CG96].

The power of this logic-based formalism was investigated in [CV99] and [Got94a, Got94b] using different variations of doughnut-shaped regions. Bennett et al [BCTH00] used this connection logic to describe qualitative geometry. However, it was noted in [Coh00] that the region connection method is limited to studying objects of similar dimension and can't handle objects with different dimensions. Theorem proving techniques were used for reasoning (static behaviour modeling) on the defined axioms. The difficulties and complexity of such task are reported by Cohn in [RCC92b].

In the constraint-driven approach, an object is represented in terms of the set of its components, and relationships are the result of the combinatorial intersection of those components. These group of approaches generally aim to satisfy completeness, but not soundness. The degree of completeness is dependent on the modeling strategy adopted for space and relations.

An intersection-based approach was proposed by Egenhofer et al [EGG$^+$99, EF91] where point-set topology was used for the definition of the components of two simple regions as interior (A°), boundary (δA) and exterior (A^-). Spatial relationships between the regions considered are the result of the exhaustive combinatorial intersection of their components ($2^9 = 512$ possible relationships in this case). Only eight relations between the regions are possible. Special rules were introduced to reduce the combinatorial set and eliminate the non-sound relations. A set of 8 rules were used to reason about the relations between the regions. The rules are, however, specific and could be applied only between simple regions.

Various extensions of this approach has been proposed to represent relationships between lines [Ege93], between regions with holes [ECP94] and between regions with indeterminate boundaries [CDF96b]. Clemintini and De Felice [CDF01b] have further extended the later model to handle complex objects with broad boundaries, where they identified 56 possible relations in this case.

All the identified sets of relations above can be categorised as set M_2 in figure 8.2. Egenhofer's approach and the various extensions thereof are limited, as specific soundness rules have to be devised to eliminate invalid relations on a case by case basis and whenever a new type of object is considered. Furthermore, the reasoning rules proposed were limited to the case of simple convex regions only.

Hence, while the constraint-driven approach guarantee completeness, it does not provide for soundness of representation, and the relation-driven approach is sound but not complete. Note that if the domain is restricted to a set of simple (regular) shapes then both approaches can be made complete and sound with respect to representation.

In a previous chapters, an intersection model was proposed that generalised the representation of objects and spatial relations. It was shown how the model can apply to objects with arbitrary complexity and to model their static behaviour. The model was extended to handle orientation and proximity, temporal and spatio-temporal relations. The approach represents relations set C in figure 8.2. Although complete and general, the model had to assume that the set of sound relations between any of the considered objects were pre-defined.

The only practical method for reducing the set C to the complete and sound set A^+ is to devise general soundness rules that are applicable on spatial relations between any type of spatial objects. This chapter addresses this problem by studying the characteristics of the underlying qualitative space as explained below.

8.1 General Soundness Rules and Constraints

To reduce the set of complete relations in a domain to the set of complete and sound ones, soundness rules have to be devised which incorporate the physical properties or constraints of the topological space. Relations that do not conform to those rules would thus be filtered out. Physical properties of the topological space are constant under any topological mapping. Hence, properties such as size and shape are not considered. The set of soundness rules, denoted here as topological mapping rules, are then transformed to a set of constraints that can be directly applied to reduce the combinatorial intersection in the intersection matrices to only those representing physically possible or sound relationships.

The set of rules and constraints represents the properties most commonly used by humans in the process of topological visual reasoning. The representation formalism together with the constraints proposed here represent a major step towards developing a general theory for qualitative space.

8.1.1 General Topological Mapping Rules

In a topological space, object properties remain invariant under topological transformations such as, stretching or rotation. The following set of rules captures the main characteristics of the qualitative space and govern the process of space and object decomposition in that space.

Connectivity Rule: A connected component x will preserve its connectivity under any topological transformation.

Component Dimension Rule: A component x with dimension n, where $n = 0 \vee 1 \vee 2 \vee 3$ will preserve its dimension under any topological transformation.

Closed Component Rule: A closed component, e.g. a line forming a closed curve or an area forming a closed surface, will preserve its closure under any topological transformation.

Closure Rule: An open set will remain open under any topological transformation and a closed point set will remain closed under any topological transformation.

In addition, the two assumptions used in the model, those of infinity and equality of spaces have to be preserved under topological transformations. The following two rules captures both assumptions.

Infinite Component Rule: Infinite components of a space will remain infinite under any topological transformation.

Space Equality Rule: Any two infinite and equal spaces will remain equal under any topological transformation.

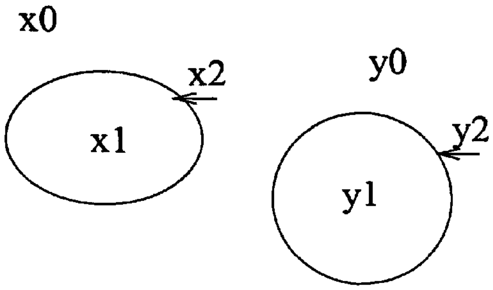
8.1.2 Mapping Rules to Soundness Constraints

The above rules must apply when objects interact in any possible topological relationship. Hence, any intersection relation that violates one or more of the above rules is not a physically possible relation and can be omitted. The following constraints are the interpretation of the above rules on the intersection relations. The simple case of the intersection relations between two simple regions x and y in 2D space is used here to illustrate the concepts.

a. **Connectivity constraint:** If y_j is a connected component of space Y and x' is a subset of space X, where $x' = \{x_1, x_2, \cdots, x_{m'}\}$ and $m' \leq m$ and m is the total number of components of space x, then if $y_j \sqsubseteq x'$, x' must be connected. I.e. each component in the set x' must be adjacent to one or more components of the set x'. If x' is not connected, then the corresponding intersection relation is false.

   Hence, in the case of simple regions x and y, the matrix in figure 8.3(b) is false and can be eliminated. In the matrix $y_1 \sqsubseteq \{x_1, x_0\}$ and x_1 is not connected to x_0.

   This condition can be generalised as follows. Let $y' \sqsubseteq x'$ and $y' = \{y_1, y_2, \cdots, y_{n'}\}$ and $x' = \{x_1, x_2, \cdots, x_{m'}\}$. If x' is connected, then y' must also be connected. For example, the following matrix is not valid.



(a)

	y_1	y_2	y_0
x_1	1	-	-
x_2	0	-	-
x_0	1	-	-

(b)

x_0		
1	x_2	
0	1	x_1

(c)

y_0		
1	y_2	
0	1	y_1

(d)

Figure 8.3: (a) Simple regions x and y. (b) An impossible intersection matrix for the regions. A $-$ in the matrix represents $0 \vee 1$.

	y_1	y_2	y_0
x_1	1	0	-
x_2	0	0	-
x_0	0	1	-

In the matrix, let $y' = \{y_1, y_2\}$ and $x' = \{x_1, x_0\}$. In this case y' is connected but x' is not.

b. **Dimension constraint:**

If y_j is a component with dimension p in space Y, and $x' \subseteq X$ where $max(dim(x')) = q$, then $y_j \sqsubseteq x' \rightarrow q \geq p$.

This constraint states that an object component of a certain dimension can't be a subset of another object component of lower dimension. Hence, the following matrix is false in the case of two simple regions, since $y_1 \sqsubseteq x_2$ and $dim(y_1) > dim(x_2)$. The object components are as shown in figure 8.3(a).

	y_1	y_2	y_0
x_1	0	1	0
x_2	1	1	0
x_0	0	1	1

Accordingly, an object component of $dim = 0$, i.e. a point, can't have a positive intersection with more than one component of the other space of any dimension.

c. **Closure constraint:** If y_j is a closed component of space Y, i.e. a closed line or an area, and if x_i is a non-closed component of space X of the same type (i.e. x_i is a line if y_j is a line and an area if y_j is an area), then $y_j \sqsubseteq x_i$ represents a false relation.

   The reason being, if $y_j \sqsubseteq x_i$ then x_i must be either closed (a contradiction) or intersects itself (excluded case by assumption) or of higher dimension. An example of this constraint is in the case of line-region relations where if the region boundary have a positive intersection with the line, it must also intersects its embedding space as well.

d. **Open and closed set constraint:** Let y_j be a component of space Y and $x' \subseteq X$. If y_j is an open set and $y_j \sqsubseteq x'$, then x' must also be an open set. Remember that $y_j \sqsubseteq x' \rightarrow y_j$ intersects only with every member of the set x'.

   Consider the example of the two simple regions x and y. The components of space X are x_1, x_2 and x_0 as shown in figure 8.3. Either the components $x_1 \cup x_2$ are closed and x_0 is open or the components $x_0 \cup x_2$ are closed and x_1 is open. Hence, if y_1 (an open set) intersects with both x_1 and x_2, the result of the intersection must be an open set which has to consequently intersect with x_0 as well. The same is true if y_1 (or y_0) intersects x_0 and x_2. Hence, $y_1 \sqsubseteq (x_1 \cup x_2)$ is not valid, since y_1 is an open set and $x_1 \cup x_2$ is a closed set. Intuitively, this means that y_1 can't intersect with both x_1 and x_2 without part of y_1 intersecting part of x_2. Hence, any relation conforming with the following matrix is not valid.

	y_1	y_2	y_0
x_1	1	-	-
x_2	1	-	-
x_0	0	-	-

Another implication of this constraint can be stated as follows. If a component y_j of the same dimension of the embedding space intersects with a component of dimension 0, i.e. a point, it must intersect with all its adjacent components. Otherwise, it intersects its boundary whose dimension is less than the embedding space.

e. **Infinity constraint:** If y_0 is an infinite component of space X and if $y_0 \sqsubseteq x'$, then x' must contain at least one infinite component.

Intuitively this constraint says that it is impossible for an infinite component in the space to only have an intersection with finite component(s). In this case the infinite component becomes a subset of the finite component(s) which is not possible. Hence, any relation conforming with the following matrix is not valid.

	y_1	y_2	y_0
x_1	-	-	-
x_2	-	-	-
x_0	-	-	0

f. **Space equality constraint:** Every component from one space must intersect with at least one component from the other space.

If one component of one space does not intersect with any component of the other space, either the two spaces are not equal or the spaces are not dense or *connected.* Both conditions are excluded by the initial assumptions. This implies that there cannot exist a row or a column in the intersection matrix whose elements are all empty intersections. Hence the combinatorial cases in the matrix where this case exists can be ignored. For example, relations represented by the following matrix are not valid.

	y_1	y_2	y_0
x_1	0	-	-
x_2	0	-	-
x_0	0	-	-

8.1.3 Non Topological or Domain Specific Constraints

In studying specific problem domains, more specific constraints, in addition to the general ones, need to be identified and applied to filter out non-sound relations. In some cases, quantitative as opposed to qualitative properties need to be considered. Four general types of domain specific constraints can be identified.

- **Component Size/dimensions Constraint:** The size of an object component, measured by its length, width, area or volume plays a role in filtering out invalid relations where components of larger size cannot be subsets of components of smaller (shorter, narrower) size. If $y_j > x_i$ then any relation where $y_j \sqsubseteq x_i$ is an invalid relation.
- **Component Shape Constraint:** This constraint excludes the cases where two components of different shapes intersect only with each other. If x_i and y_j are two components of spaces x and y respectively, then if the shape of y_j is not equal to the shape of x_i, y_j can't be equal to x_i. I.e. $y_j \sqsubseteq x_i$ and $x_i \sqsubseteq y_j$ are false relations.
- **Physical Properties Constraint:** Many different types of constraints related to the physical properties of the objects studied may be used, such as permeability, rigidity, elasticity, deformability, etc. Considering those constraints may lead to the elimination of cases where some interaction between the components of the different spaces are not allowed. For example, a rigid component of one space can only intersect with the complement of the surrounding space of a non-permeable object.
- **Spatial Arrangement Constraint:** This constraint involves the identification of sound relations based on the allowable spatial arrangements of different object components, using orientation and relative distance relations.

The computation is usually simpler and more effective if domain-specific rules were applied first to eliminate some non-sound relations. Those constraints can significantly reduce the

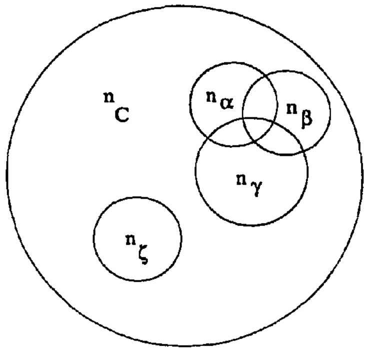


Figure 8.4: Overlapping effect of the soundness rules.

number of possibilities studied. General soundness constraints can be applied later to produce the set of complete and sound set of relations.

8.2 Calculating the number of Complete and Sound Relationships

Let the set $R = \{\alpha, \beta, \gamma, \zeta\}$ be the set of soundness constraints that will be applied to an intersection matrix of N elements. The total number of complete relations n_C is 2^N relations.

Let n_f be the total number of false relations excluded by the set of constraints R. Hence, the set of sound relations n_S is defined as: $n_S = n_C - n_f$. If $n_\alpha, n_\beta, n_\gamma$ and n_ζ are the sets of relations eliminated directly by the application of the constraints $\alpha, \beta, \gamma, \zeta$ respectively, there is no guarantee that there will be no overlap between the sets of relations excluded by each constraint. In this case, the overlap between constraints has to be accounted for to ensure that some combinations are not excluded more than once from the whole set n_C.

The number of false relations can therefore be calculated as follows:

$$n_f = (n_\alpha + n_\beta + n_\gamma + n_\zeta) - (n_{\alpha\beta} + n_{\alpha\gamma} + n_{\alpha\zeta} + n_{\beta\gamma} + n_{\beta\zeta} + n_{\gamma\zeta})$$

$$\begin{aligned} &+ \quad (n_{\alpha\beta\gamma} + n_{\alpha\beta\zeta} + n_{\alpha\gamma\zeta} + n_{\beta\gamma\zeta}) - n_{\alpha\beta\gamma\zeta} \\ &= \quad A - B + C - D \end{aligned}$$

where

- A = the total number of relations in the set α, β, γ and ζ.
- B = the number of relations resulting from the intersection of each two sets of α, β, γ and ζ.
- C = the number of relations resulting from the intersection of each three sets of α, β, γ and ζ.
- D = the number of relations resulting from the intersection of all the sets α, β, γ and ζ.

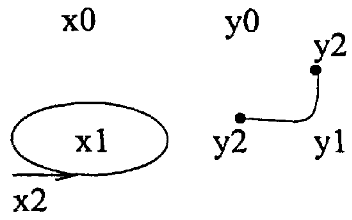
Two constraints sets overlap if their common elements in the intersection matrix had the same entry of 0 or 1, (e.g. $x_i \cap y_j = 0$). If their corresponding intersection result differs in one ore more element in the matrix, then they do not overlap.

8.3 Example: Determining the set of complete and sound relations between a region and a line

Consider objects x and y in figure 8.5 The number of possible instances of different intersection matrices for those objects is equal to 2^{n+m}, i.e. 2^9.

The application of constraints 2, 5 and 6 above results in the intersection of the component y_0 with all the components of X and consequently reduces the number of possible combinations to 2^6.

	y_1	y_2	y_0
x_1	-	-	-
x_2	-	-	-
x_0	1	1	1



(a)

	y_1	y_2	y_0
x_1	-	-	-
x_2	-	-	-
x_0	-	-	-

(b)

Figure 8.5: (a) Calculating the number of sound relationships between a region and a line. (b) Combinatorial intersections A $-$ in the matrix represents $0 \vee 1$.

The rest of the soundness constraints apply to the matrix as follows.

$I(x, y) =$

	y_2		y_1	
	0	1	0	1
x_1	(a)	(c) (e)	(b) (c)	(f)
x_2	(a)	(e)	(b) (f)	(-)
x_0	(a)	(e) (d)	(b) (d)	(f)

- a : y_2 has no intersection with space X (constraint 6)
- b : y_1 has no intersection with space X (constraint 6)
- c : $y_2 \cap x_1 = 1$ and $y_2 \cap x_2 = 1$ and $y_2 \cap x_0 = 1$. Since y_2 is a set of two separate points and x_1 is an area, x_1 must intersect with the adjacent component to y_2 which is y_1 in this case (constraint 4)
- d : $x_0 \cap y_2 = 1$ and $x_0 \cap y_1 = 0$. Similar to c (constraint 4)
- e : $x_1 \cap y_2 = 1$ and $x_1 \cap y_1 = 0$.

This is impossible since y_2 consists of two points and a point can have a positive intersection with only one component (constraint 4).

- f : $y_1 \sqsubseteq \{x_1, x_0\}$. x_1 and x_0 are not connected (constraint 1).

Taking into account the overlapping elements in the result of application of the different constraints, and noting that if two constraints imply both positive and negative intersection for the same components, then they do not overlap and can be excluded. The set of sound relations can be calculated as follows:

$$
\begin{aligned}
n_S &= n_C - (n_a + n_b + n_c + n_d + n_e + n_f) + \\
&\quad (n_{ab} + n_{af} + n_{bc} + n_{bd} + n_{be} + n_{cd} + n_{ce} + \\
&\quad n_{de} + n_{ef}) - (n_{bcd} + n_{bce} + n_{bde} + n_{cde}) + \\
&\quad (n_{bcde}) \\
&= 64 - (2^3 + 2^3 + 2^4 + 2^4 + 2^3 + 2^3) + \\
&\quad (2^0 + 2^0 + 2^2 + 2^2 + 2^0 + 2^2 + 2^2 + 2^2 + 2^0) - \\
&\quad (2^1 + 2^0 + 2^0 + 2^1) + \\
&\quad 2^0 \\
&= 64 - (64) + (24) - (6) + (1) = 19
\end{aligned}
$$

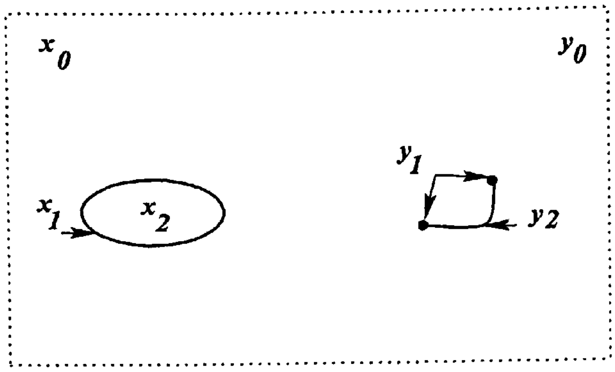
The sound and complete set of 19 relationships are shown in tables 8.6 and 8.7.

8.4 Deriving Sound Relations using Domain Specific Constraints

Consider the example shown in figure 8.8, where the relations between tennis racket and a tennis ball are considered.

Three components are used to represent the ball and a simplified representation of four components were used to define the racket. The three dimensional problem can be reduced to 2D space using the following assumptions.

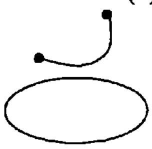
- The tennis ball is solid (not permeable) and hence can be represented as a 2D region consisting of only two components, x_0 and x_1, as shown in figure 8.8 (c).



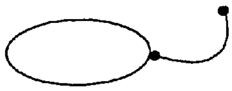
$$\begin{bmatrix} x_1 \cap y_1 & x_1 \cap y_2 & x_1 \cap y_0 \\ x_2 \cap y_1 & x_2 \cap y_2 & x_2 \cap y_0 \\ x_0 \cap y_1 & x_0 \cap y_2 & x_0 \cap y_0 \end{bmatrix}$$

(a)

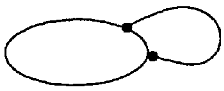
$$R_1(x,y) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



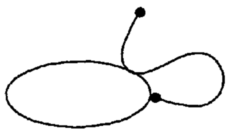
$$R_2(x,y) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



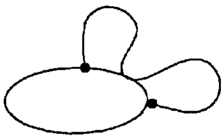
$$R_3(x,y) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$



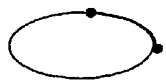
$$R_4(x,y) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



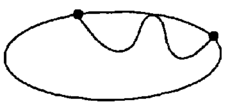
$$R_5(x,y) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$



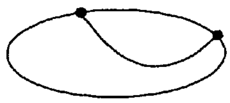
$$R_6(x,y) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$



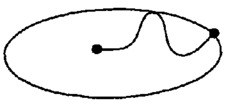
$$R_7(x,y) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$



$$R_8(x,y) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$



$$R_9(x,y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$



$$R_{10}(x,y) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

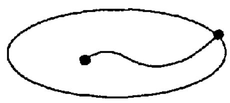
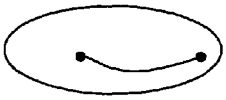
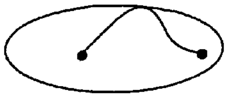


Figure 8.6: Part of the set of sound relations between a region and a line.

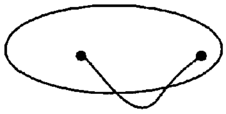
$$R_{11}(x,y) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$



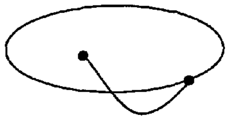
$$R_{12}(x,y) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$



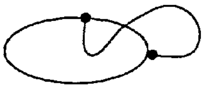
$$R_{13}(x,y) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$



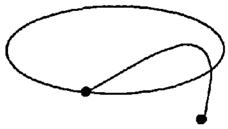
$$R_{14}(x,y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$



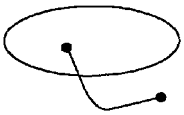
$$R_{15}(x,y) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$



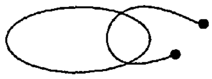
$$R_{16}(x,y) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



$$R_{17}(x,y) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



$$R_{18}(x,y) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



$$R_{19}(x,y) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Figure 8.7: The rest of the set of sound relations between a region and a line.

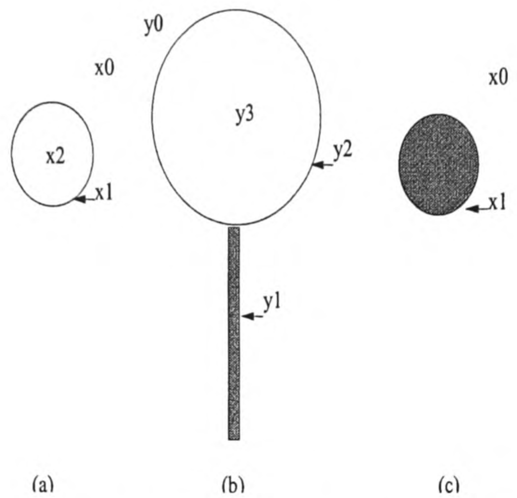


Figure 8.8: An example of using domain-specific constraints. (a) A tennis ball. (b) A tennis racket.

Note that the tennis ball shape as a sphere, combined with its rigidity implies point contact of the component x_1 with any component y_1, y_2 or y_3. However, as the ball is elastic, its intersection with components of space y will be assumed to be an area.

- No level difference exist between the frame of the tennis racket and the racket net which eliminates the need for further components.
- The racket is symmetrical around the proposed 2D plane.

The intersection matrix representing the possible relations is defined as follows, giving rise to $2^8 = 256$ possible relations.

	y_1	y_2	y_3	y_0
x_1	-	-	-	-
x_0	-	-	-	-

A second domain-specific constraint Φ is that the size of the tennis ball is smaller than the size of the racket net. Hence, $y_3 \not\sqsubseteq x_1 \rightarrow x_0 \cap y_3 = 1$. From constraint number 6, we have $x_0 \cap y_0 = 1$. Hence, $x_0 \sqsubseteq \{y_3, y_0\}$. Since, y_3 and y_0 are not connected, hence $x_0 \cap y_2 = 1$, i.e. x_0 must intersect with y_2 as well.

A further constraint is that of size, where the diameter of the tennis ball is smaller than the length of the tennis racket handle y_1. Hence, $y_1 \not\subseteq x_1 \rightarrow y_1 \cap x_0 = 1$.

Accordingly, the intersection relation of the component x_0 with all the components of space y are positive. The remaining intersection relations are between x_1 and space Y, namely, $2^4 = 16$ relations.

The significant reduction in the number of cases considered demonstrates the benefits of prior application of domain-specific constraints in the process of eliminating invalid relations.

The general soundness constraints apply to the matrix as follows.

$I(x, y) =$

	y_1		y_2		y_3		y_0	
	0	1	0	1	0	1	0	1
x_1	(b) (e)	(c) (d)	(a) (c) (e)	(b)	(b) (e)	(a) (c)	(e) (b) (d)	(a)

- a : x_1 intersects with y_3 and y_0 but not with y_2 (connectivity constraint)
- b : x_1 intersects only with y_2 (dimension constraint)
- c : x_1 intersects with y_1 and y_3 but not with y_2 (connectivity constraint)
- d : x_1 intersects with y_1 but not with y_0 (dimension constraint)
- e : x_1 has no intersections (space equality constraint)

The set of sound relations can be calculated as follows:

$$\begin{aligned} n_S &= n_C - (n_a + n_b + n_c + n_d + n_e) + \\ &\quad (n_{ac} + n_{cd}) \\ &= 16 - 10 + 2 = 8 \end{aligned}$$

The eight relations are shown in figure 8.9. The corresponding possible intersections of the component x_1 with the space Y is as follows: 1) $(0,0,0,1)$, 2)$(0,1,0,1)$, 3)$(0,1,1,1)$,

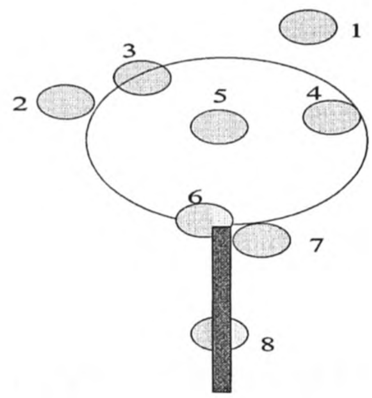


Figure 8.9: The set of eight sound relations between a tennis ball and a tennis racket.

4) $(0,1,1,0)$, 5) $(0,0,1,0)$, 6) $(1,1,1,1)$, 7) $(1,1,0,1)$ and 8)$(1,0,0,1)$.

8.5 Conclusions

General rules are proposed for the derivation of the set of sound qualitative spatial relations between objects of arbitrary complexity. They are based on the topologically invariant aspects of space. The rules are mapped into general constraints governing different aspects of space and object representation, including connectivity, component dimension, closed components, open sets, infinite sets and space equality. The constraints proposed complements a general formalism for qualitative spatial representation and reasoning proposed earlier [BA97] and together provide means for the automation of spatial reasoning techniques and their implementation. Domain specific constraints can be specified to reflect the characteristics of different problems. A method of calculating the number of sound relationships is also presented.

Chapter 9

Conclusions and Future Work

In this thesis an approach to qualitative spatial and spatio-temporal representation and reasoning was developed. Three main objectives were set for the work, all related to the current challenges facing this research area today. First, the need to address the trade-off between expressiveness and the reasoning ability of developed formalisms. Secondly, to develop QSRR formalisms which are both complete and sound. Thirdly, is to extend the depth (ontological coverage) and breadth (domain) of expressiveness of the developed methodologies. The latter issue is concerned with providing a homogeneous treatment for the spatial and the spatio-temporal domains, e.g. by using a homogeneous ontological base of primitive entities and modeling space.

9.1 Summary and Conclusions

The primary objective of this thesis is to address two main challenges facing the development of general formalism for qualitative spatial and spatio-temporal representation and reasoning. The first challenge is the trade-off between completeness and soundness of the representation and reasoning formalisms. Most works in this field can be categorised as biased to one of those qualities. Thus, works would prioritise the satisfaction of soundness or completeness, and then seek to prove or satisfy the other measure.

The second challenge is the trade-off between expressiveness and efficiency or reasoning power. This trade-off stems from the fact that while extending the representation models to handle more complex spatial scenes, new concepts or axioms need to be introduced. The addition of more concepts further complicates the reasoning mechanisms.

A study of related work in the area of QSRR resulted in the classification of approaches to three general categories: those which model objects in space with lower dimensions, namely, projection-based methods, and those which uses modeling spaces of dimension equal to or higher than the objects' dimension. The later approaches are further divided into point-set based approaches and region-based approaches.

A point-set intersection-based approach is proposed for the representation and reasoning in spatial and spatio-temporal domains.

The method is based on the decomposition of the objects and their embedding spaces into components of interest to the application considered. The topology of the object and space components are captured in a structure, denoted the "adjacency matrix", that records the adjacency relationships between the different components of the objects involved. The spatial relationships between the objects is represented through the intersection relationships between the components of both spaces. Reasoning over spatial relationships is carried out by propagating the empty and non-empty intersection relationships using two general reasoning rules.

The following is a summary of the main conclusions of this thesis, stated from the point of view of satisfying the objectives set out for the project.

I. Expressiveness

The following are the major contributions towards achieving expressiveness.

- The adjacency matrix proposed for representing the topology of objects and their embedding space is a general structure. It can represent objects with arbitrary complexity. The structure is versatile where different levels of representation detail can be used. No restrictions are made on either the type of objects represented nor on their complexity, i.e. the number of components making up the objects and the space. Earlier works representing objects by their exterior, boundary and exterior are not adopted.

- An intersection matrix is proposed for representing spatial relations between objects of arbitrary complexity. The matrix gives exhaustive and pairwise disjunctive EPWD representation of relations, which is considered to be complete under the assumption of similar relations-similar intersection matrices. No restriction is imposed on the type, complexity or dimension of the interacting objects.

- The method was used to represent composite regions (composed of non-connected object parts) and to configurations where virtual parts such as semi-closed finite parts of space or non-physical components such as magnetic fields or temperature contours are represented.

- Different types of spatial relations, namely orientation and proximity relationships are shown to be represented using the proposed formalism in a coherent and homogeneous fashion and without the need to introduce any new concepts.

- The representation of temporal spaces and temporal relations was shown to be possible using the adjacency and intersection structures. The method was therefore shown to extend naturally to the spatio-temporal domain.

- Inaccuracy and uncertainty are characteristics of many applications of spatial domains. In this thesis, the concepts of uncertainty is studied in detail. Four types of uncertainty are identified, namely, position, extension, configuration and orientation. The representation formalism was extended to cater for the different types of uncertainty by allowing for the disjunctive labelling of object and space components. Two modes of uncertainty were also defined and represented. Discrete uncertainty where the set of labels used do not represent adjacent components and range (or continuous) mode where the set of labels represent adjacent components. Partial uncertainty was also identified where the disjunctive set of labels does not apply to whole components.

II. Completeness-Soundness Challenge

The method developed in this thesis is a completeness-first approach, i.e. the method is guaranteed to be complete. To achieve soundness, rules to constrain and filter the complete set of relations into a sound set of relations needed to be devised. In other works, proposed soundness rules were specific to particular spatial configurations. However, for the method to be general, the rules must be independent of any constraint, including, object type, complexity and dimension as well as relationship type.

- The topological invariants, preserved under topological transformations were used as basis for the proposed rules. The invariants capture connectivity, dimenstionality, closure of component or set as well as the assumptions of infinity and equality of embedding spaces.
- Domain specific constraints such as the size and shape of objects can influence the representation of objects and the possible relationships between objects. The effects of such constraints as well others, for example, the permeability of the object and how the object is situated in terms of its orientation was studied.
- A mathematical calculation of the number of sound relations was developed based on the general and domain-specific soundness rules.

III. Expressiveness-Efficiency Challenge

The issue of expressiveness was addressed above by providing a general ontology of space that is expressive enough to represent different models of reality in the spatial and spatio-temporal domains. To address the expressiveness versus efficiency challenge, any spatial reasoning mechanisms proposed must be based on and utilise the provided ontology. The reasoning approach proposed in this thesis is summarised below.

- The proposed reasoning rules are based on checking the empty and the non-empty intersections of object and space components and are therefore completely confined within the scope of the representation scheme and requires no extensions or new concepts. The rules are complemented by general constraints on the equality and infiniteness of the spaces.
- The propagation of spatial relations using the reasoning rules involves a number of steps equal to the number of components on the objects involved. Hence, the reasoning problem is always solvable in a finite number of steps.
- Completeness and soundness of the rules were proved with respect of set theory.
- An analysis of the reasoning process helped in understanding some interesting issues related to the propagation of definite, and indefinite intersection, as well as propagation of no new knowledge in some cases.
- Spatial reasoning with incomplete knowledge has been addressed and reasoning of disjunctive sets of relations was simplified. The reasoning rules were also generalised to be applied in the absence of composition tables.
- Finally, the reasoning formalism has been implemented as a prototype engine using Java, and was used to derive a variety of composition tables, including the composition table (44×44 entries) between regions with undetermined boundaries.

Limitations of the proposed formalisms

The representation approach doesn't apply to the following situations.

a. Topological relations between objects with overlapping components.

b. Orientation relations between composite objects or between objects with distorted shapes such as concave regions in close proximity.

c. Representation and fine distinction between different types of overlap or inside relations.

d. Spatio-temporal relations between complex spatio-temporal objects such as composite or branching objects.

e. The completeness of the soundness rules is yet to be proven. Their soundness is derived from the soundness of the topological invariants they present.

In terms of reasoning, the following issues are not yet addressed in this work.

a. Hybrid reasoning that combines different types of topological, orientation and proximity relations.

b. Constraint satisfaction (static reasoning) in spatial and the spatio-temporal domain was not addressed.

c. The automation of the derivation of the transition graph (dynamic reasoning) was not addressed.

No account was taken of the size or shape of the objects as such object characteristics do not have an impact of the components of the objects or their adjacency. This is a general limitation to other qualitative approaches in the literature as well.

9.2 Recommendations for Future Work

The limitations listed above set out the scene for future extensions and developments on the work in this domain. The following are possible extension themes for the work described in this thesis.

- The method developed has been applied on different types of relations individually. Heterogeneous reasoning by combining the treatment of more than one type of spatial relationship is needed.
- A systematic study and classification of different spatio-temporal relationships is needed, in the same fashion as has been done for space.
- A more elaborate study of the issue of soundness is required and the soundness rules identified need to be validated for completeness.
- A study of the characteristics, complexity, costs and gains of the spatial algebra for reasoning with uncertainty need to carried out.
- Extending the reasoning method by investigating its application in the dynamic reasoning domain. The key to such an extension is the relation between the conceptual neighbourhood relations and the adjacency of the intersecting components.
- Investigating the impact of applying the general reasoning equation developed for incomplete knowledge in the domain of constraint satisfaction.
- Actual application of the method by developing a qualitative engine for supporting large spatial databases is an interesting subsequent challenge to utilise the theories developed to date.
- The incorporation of combined quantitative and qualitative aspects of space needs to be studied further to build upon the results achieved in the pure qualitative domain.

Finally, it is envisaged that the work proposed in this thesis can be of benefit in many domains. In particular, the reasoning engine can be used for the automatic derivation of useful composition tables. Spatial reasoning rules can be applied to check the similarity of spatial configurations and scenes as well an in maintaining the integrity of large spatial databases.

Appendix A

SPARQS: A Spatial Reasoning Engine for Qualitative Spaces

To demonstrate the validity of the proposed approach, a reasoning engine has been designed and implemented using java. The interface to the program, named SPARQS (SPAtial Reasoning in Qualitative Space) consists of two parts. A basic interface is provided, where the topology of some common spatial object shapes are predefined, as shown in figure A.1(a). Users are able to choose object types from a menu of available ones, namely, points, line, simple region, region with indeterminate boundaries and concave regions. Users are then offered a selection of possible topological spatial relationships between the chosen object types, Sets of relationships are shown graphically and categorised using a coarse classification scheme under four headings, namely, disjoint, inside, overlap and touch to enhance the usability of the interface. The reasoning rules are applied to propagate the intersection matrices and produce the result matrix. The constraints in the matrix are then matched to the set of possible relationships and all the ones satisfying the constraints are displayed in the result window, as shown in figure A.1(a) . The program is flexible where the input spatial relationships can be changed and resubmitted and the result re-calculated, as shown in figure A.1(b).

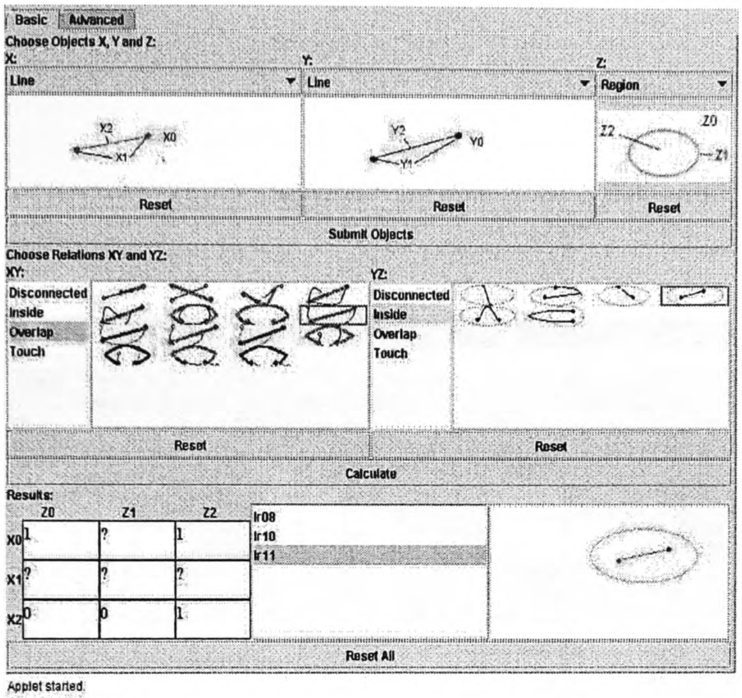
A preliminary implementation of an advanced interface is also provided as shown in figure

A.2. The intention is for users to be able to fill in adjacency and intersection matrices, which are subsequently used by the system to derive the resulting relationships. Some validation checks are done on the input matrices, e.g. to reject matrices that violate the general constraints described earlier, where no rows or columns in the matrix are allowed to contain only zeros. The result constraint matrix is therefore dependent on the validity of the input shapes and relations. Enhancement to the interface may be possible, where a more guided approach to input, possibly using sketch-based techniques, can be utilised to ensure valid entries.

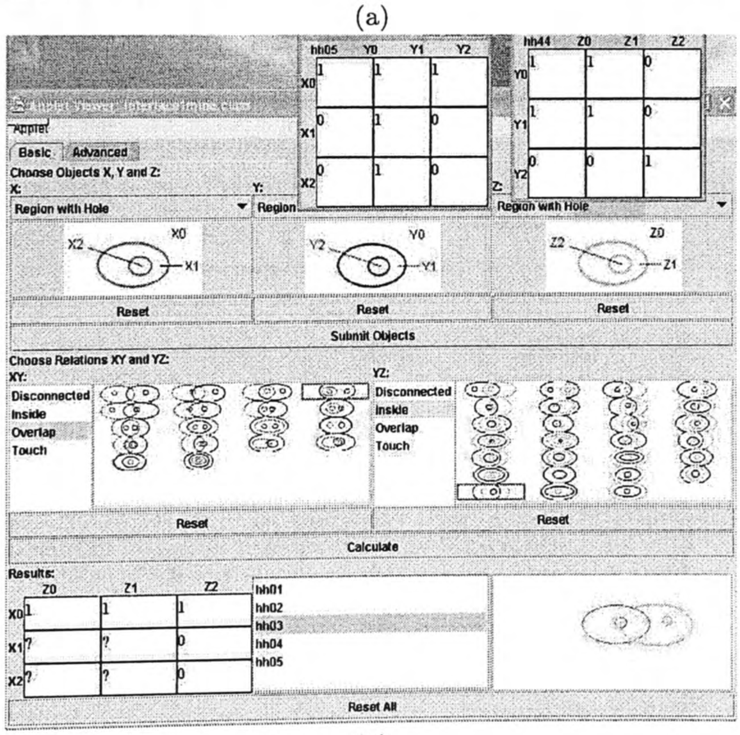
The engine has been used to derive new composition tables between all the combination of objects defined in the basic interface, e.g. between simple regions, concave regions and regions with indeterminate boundaries, etc. Part of the composition table between regions with indeterminate boundaries is shown in tables A.2, A.3 and A.4. The full set of 44 sound relations between those regions are as defined in [WC01] and are shown in table A.1.

The algorithm implementing the reasoning rules is as follows.

```
Define Intersection Matrix(x,y) and Intersection Matrix(y,z)
Find non-empty intersections (both definite and indefinite)
in Result Matrix
Find Empty intersections in Result Matrix
Map the Result Matrix to set of sound Relations
Display the resulting relations
```



(a)



(b)

Figure A.1: The basic interface in SPARQS. (a) Composition of relationships between lines and region. (b) Composition of relationships between regions with holes.

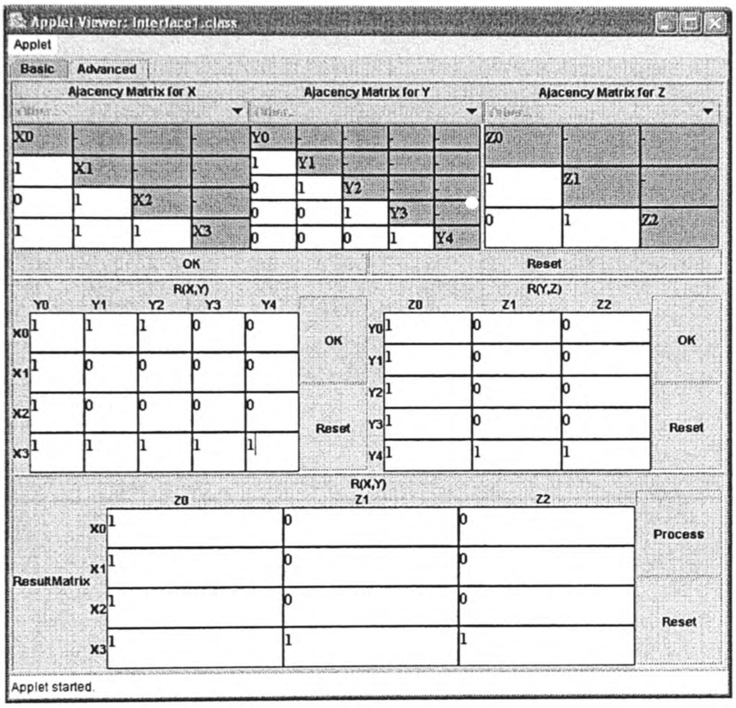


Figure A.2: The advanced interface in SPARQS. Users specify the adjacency and intersection matrices.

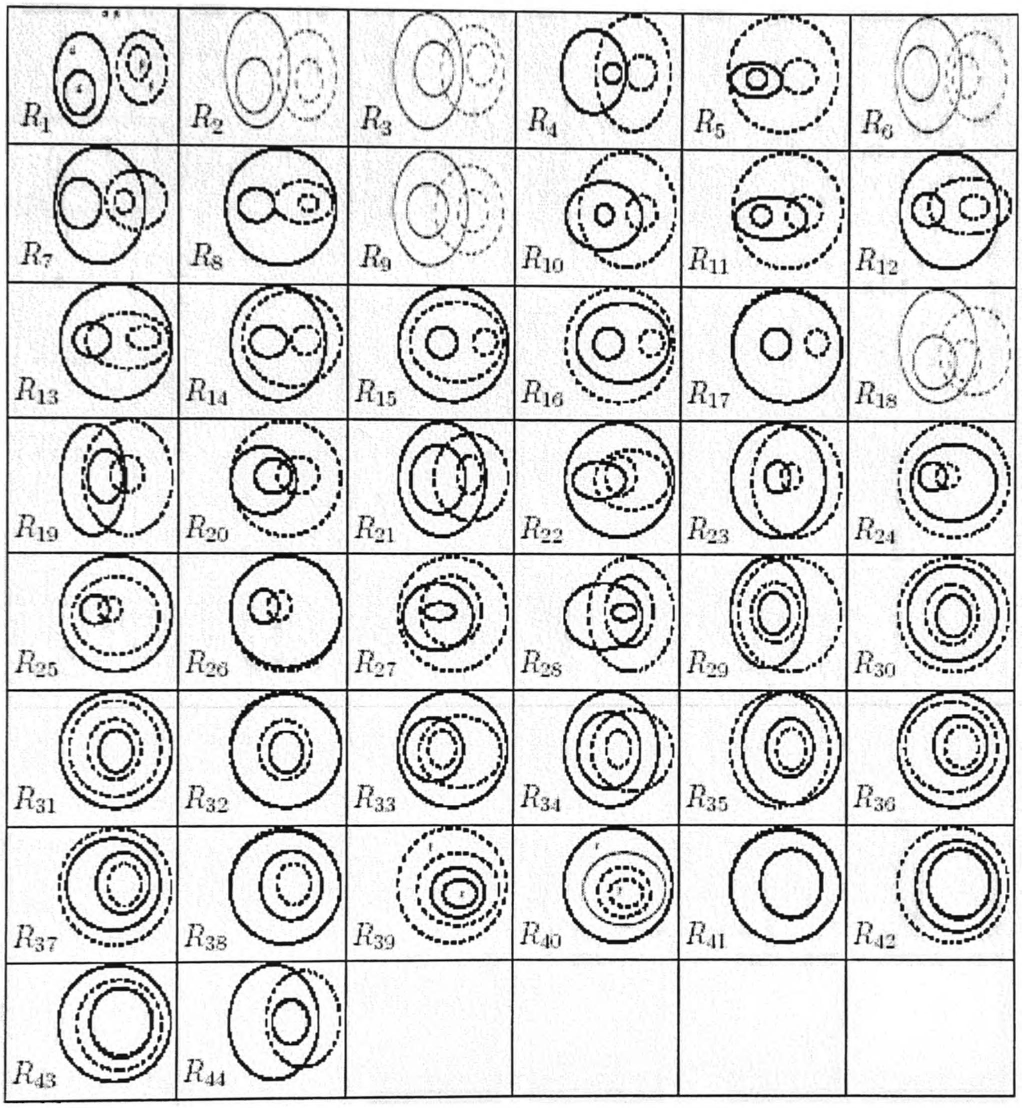


Table A.1: The set of 44 sound topological relations between regions with broad boundaries as defined in [CDF01a].

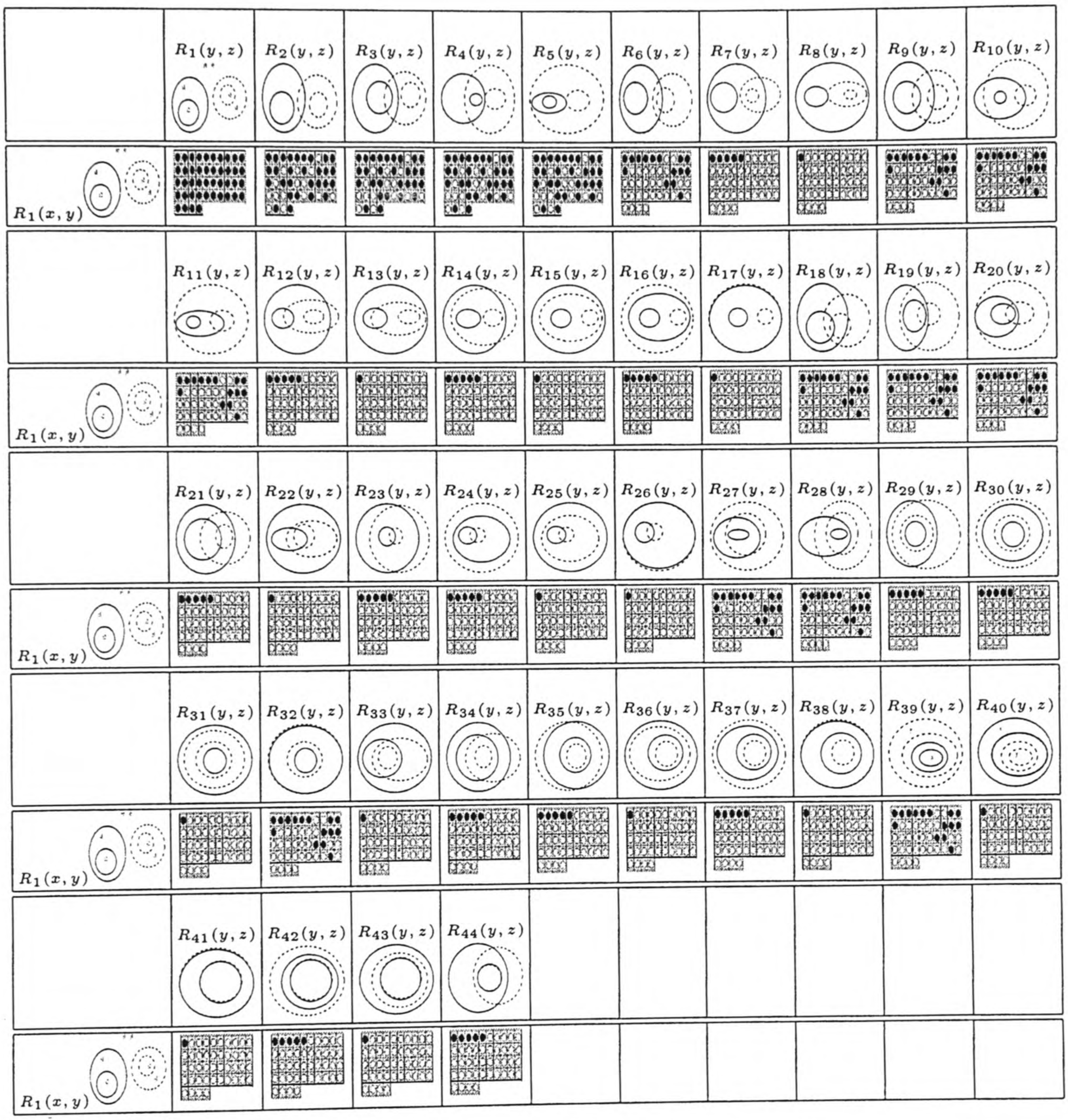


Table A.2: Part of the composition table between two regions with broad boundaries. The relations between R_1 and all the other 43 relations is shown.

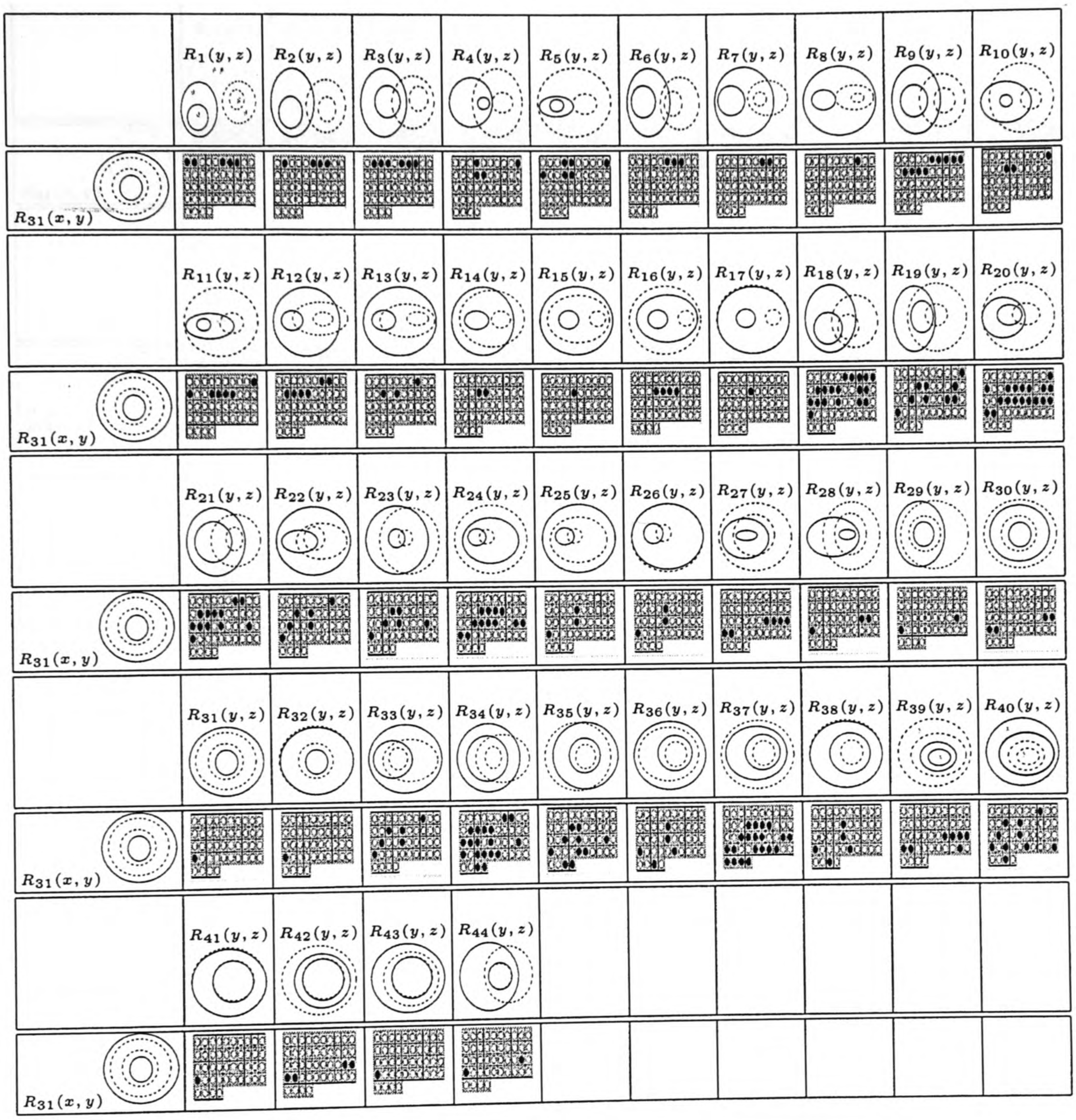


Table A.3: Part of the composition table between two regions with broad boundaries. The relations between R_{31} and all the other 43 relations is shown.

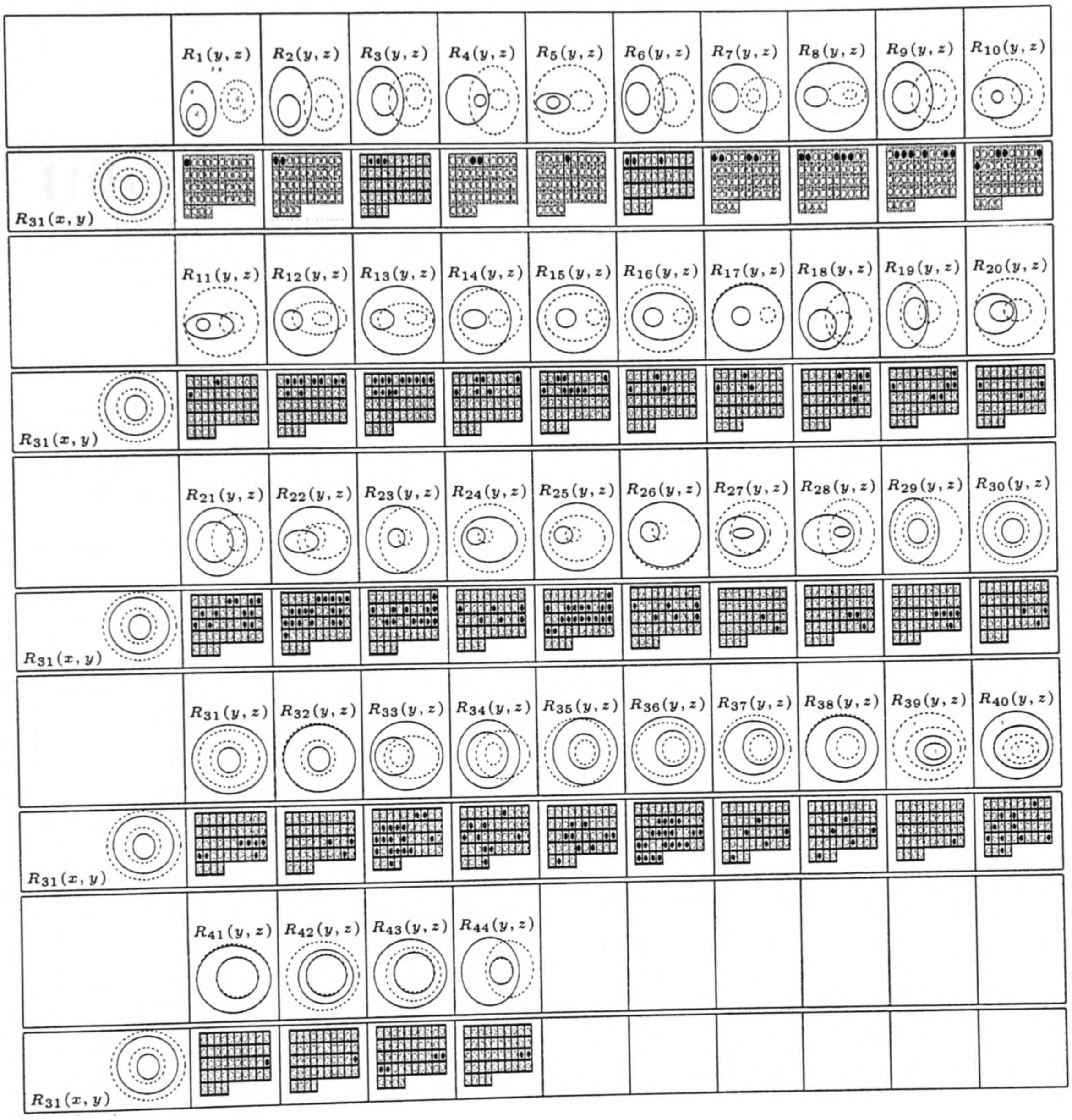


Table A.4: Part of the composition table between two regions with broad boundaries. The relations between R_{30} and all the other 43 relations is shown.

Bibliography

[AA95] A. I. Abdelmoty and El-Geresy B. A. A General Method for Spatial Reasoning in Spatial Databases. In *Proceedings of the Fourth International Conference on Information and Knowledge Management (CIKM'95)*, pages 312–317. ACM, 1995.

[All83] J.F. Allen. Maintaining Knowledge about Temporal Intervals. *Artificial Intelligence and Language Processing, Communications of the ACM*, 26:832–843, 1983.

[APW+94] Alia I. Abdelmoty, Norman W. Paton, M. Howard Williams, Alvaro A.A. Fernandes, Maria L. Barja, and Andrew Dinn. Geographic Data Handling in an Deductive Object-Oriented Database. In D. Karagiannis, editor, *Proc. 5th International Conference on Database and Expert Systems Applications (DEXA)*, pages 445–454. Springer Verlag, Sep 1994.

[BA97] El-Geresy B.A. and Abdelmoty A.I. Order in Space: A General Formalism for Spatial Reasoning. *Int. J. on Artificial Intelligence Tools*, 6(4):423–450, 1997.

[BCTH00] B. Bennett, A.G. Cohn, P. Torrini, and S.M. Hazarika. A Foundation for Region-Based Qualitative Geometry. In *Proc. 14th European Conf. on AI (ECAI-2000)*, 2000.

[CBGG97] A.G. Cohn, B. Bennett, J. Gooday, and N.M. Gotts. Qualitative Spatial Representation and Reasoning with the Region Connection Calculus. *Geoinformatica*, 1(3):1–42, 1997.

[CCB00] M. Cristani, A.G. Cohn, and B. Bennett. Spatial Locations via mopho-Mereology. In *KR 2000*, pages 15–25, 2000.

[CCR93] Z. Cui, A.G. Cohn, and D.A. Randell. Qualitative and Topological Relationships in Spatial Databases. In *Design and Implementation of Large Spatial Databases, third Symposium SSD '93*, LNCS 692, pages 396–315. Springer Verlag, June 1993.

[CD97] E. Clementini and P. DiFelice. Approximate Topological Relations. *International Journal of Approximate Reasoning*, 16:173–204, 1997.

[CDF96a] E. Clementini and P. Di Felice. An Algebraic Model for Spatial Objects with Indeterminate Boundaries. In P.A. Burrough and A.U. Frank, editors, *Geographic Objects with Indeterminate Boundaries, GISDATA*, pages 155–169. Taylor & Francis, 1996.

[CDF96b] E. Clementini and P. Di Felice. An Algebraic Model for Spatial Objects with Undetermined Boundaries. In *GISDATA Specialist Meeting: Spatial Conceptual Models for Geographic Objects with Undetermined Boundaries.* Taylor & Francis, 1996.

[CDF97] E. Clementini and P. Di Felice. Approximate Topological Relations. *Int. J. of Approximate Reasoning*, 16:173–204, 1997.

[CDF01a] E. Clementini and P. Di Felice. A Spatial Model for Complex Objects with a Broad Boundary Sypporting Queries on Uncertain Data. *DKE*, 37(3):285–305, 2001.

[CDF01b] E. Clemintini and P. Di Felice. A Spatial Model for Complex Objects with Broad Boundary supporting Queries on Uncertain Data. *Data And Knowledge Engineering*, 2001.

[CDFC95] E. Clementini, P. Di Felice, and G. Califano. Composite Regions in Topological Queries. *Information Systems*, 20(7):579–594, 1995.

[CDO93] E. Clementini, P. DiFelice, and P.V. Oosterom. A Small Set of Formal Topological Relationships suitable for End-User Interaction. In *Design and Implementation of Large Spatial Databases, third Symposium SSD '93*, LNCS 692, pages 277–295. Springer Verlag, June 1993.

[CG94] A.G. Cohn and N.M. Gotts. Expressing Spatial Vagueness in Terms of Connection. In *GISDATA Specialist Meeting: Spatial Conceptual Models for Geographic Objects with Undetermined Boundaries.* Taylor & Francis, 1994.

[CG96] A.G. Cohn and N.M. Gotts. The "Egg-Yolk" Representation of Regions with Indeterminate Boundaries. In P.A. Burrough and A.U. Frank, editors, *Geographic Objects with Indeterminate Boundaries, GISDATA*, pages 171–187. Taylor & Francis, 1996.

[CH01] A G Cohn and S M Hazarika. Qualitative spatial representation and reasoning: An overview. *Fundamenta Informaticae*, 46(1-2):1–29, 2001.

[CJ00a] C. Claramunt and B. Jiang. A Representation of Relationships in Temporal Spaces. In *Innovations in GIS VII: GeoComputation*, pages 41–53, 2000.

[CJ00b] C. Claramunt and B. Jiang. Hierarchical Reasoning in Space and Time. In *9th Int. Symposium on Spatial Data Handling, SDH00*, pages 31 41–51, 2000.

[CJ01] C. Claramunt and B. Jiang. An Integrated Representation of Spatial and Temporal Relationships. *Geographical Systems*, 3(4):154–159, 2001.

[Cla81] B.L. Clarke. A Calculus of Individuals Based of 'Connection'. *Notre Dame Journal of Formal Logic*, 22(3):204–218, 1981.

[Cla00] C. Claramunt. Extending Ladkin's Algebra on non-convex intervals towards an Algebra of Union of Regions. In *Eighth ACM GIS Int. Symposium*, pages 9–14, 2000.

[Coh95] A.G. Cohn. A Hierarchical Representation of Qualitative Space based on Connection and Connectivity. In *Spatial Information Theory, A Theoretical Basis for GIS : European Conference, COSIT'95*, pages 311–326. Springer Verlag, 1995.

[Coh00] A.G. Cohn. QSR: An Overview. *Geoinformatica*, 2000.

[Coh01] A.G. Cohn. Formalising Bio-Spatial Knowledge. In *Formal Ontology in Information Systems, Proc. Int.*, pages 198–209, 2001.

[CRC94] A.G. Cohn, D.A. Randell, and Z. Cui. Taxonomies of Logically Defined Qualitative Spatial Relations. In *Formal Ontology in Conceptual Analysis and Knowledge Representation- 94*. Kluwer, 1994.

[CRCB93] A.G. Cohn, D.A. Randell, Z. Cui, and B. Bennet. Qualitative Spatial Reasoning and Representation. In P. Carrete and M.G. Singh, editors, *Qualitative Reasoning and Decision Technologies*, pages 513–522, 1993.

[CT95] C. Claramunt and M. Theriault. Managing time in gis: An event-oriented approach. In *Recent Advances on Temporal Databases*. Springer Verlag, 1995.

[CV99] A.G. Cohn and A.C. Varzi. Modes of Connection: A Taxonomy of Qualitative Topological Relations. In *Proceeding of the International Conference on Spatial Information Theory COSIT'99*, volume LNCS 1329, pages 299–314. Springer Verlag, 1999.

[CYDA88] S.K. Chang, W. Yan, D.C. Dimitroff, and T. Arndt. An Intelligent Image Database System. *IEEE Transactions on Software Engineering*, 14(5):681–688, 1988.

[ECP94] M.J. Egenhofer, E. Clementini, and Di Felicem P. Topological Relations Between Regions With Holes. *Int. J. Geographic Information Systems*, 8(2):129–142, 1994.

[EF91] M.J. Egenhofer and R.D. Franzosa. PointSet Topological Spatial Relations. *Int. J. Geographic Information Systems*, 5(2):161–174, 1991.

[EFJ89] M.J. Egenhofer, A.U. Frank, and J.P. Jackson. A Topological Data Model for Spatial Databases. In A. Buchmann and O. Gunther, editors, *Design and Implementation of Large Spatial Databases, First Symposium SSD '89*, Lecture Notes in Computer Science 409, pages 271–286. Springer-Verlag, 1989.

[Ege91] M.J. Egenhofer. Reasoning About Binary Topological Relations. In O. Gunther and H.J. Scheck, editors, *Advances in Spatial Databases, 2nd Symposium, SSD'91*, Lecture Notes in Computer Science, 525, pages 143–161, Zurich, Switzerland., 1991. Springer-Verlag.

[Ege93] M.J. Egenhofer. Definitions of Line-Line Relations for Geographic Databases. *Data Engineering*, 16(3):40–45, 1993.

[EGG+99] M.J. Egenhofer, J. Galsgow, O. Gunther, J. Herring, and D. Peuquet. Progress in Computational Methods for Representing Geographic Concepts. *International Journal of Geographical Information Science*, 13(8):775–796, 1999.

[EH90] M.J. Egenhofer and J.R. Herring. A Mathematical Framework for the Definition of Topological Relationships. In *Proceedings of the 4th international Symposium on Spatial Data Handling*, volume 2, pages 803–13, 1990.

[EM95] M.J Egenhofer and D. Mark. Modeling conceptual neighbourhoods of topological line-region relations. *Int. Journal of GIS*, 9(5):555–565, 1995.

[ES93] M.J. Egenhofer and J. Sharma. Topological Relations between Regions in R^2 and Z^2. In *Design and Implementation of Large Spatial Databases, third Symposium SSD '93*, LNCS 692, pages 316–363. Springer Verlag, June 1993.

[ES97] M. Erwig and M. Schneider. Vague Regions. In *Proc. of the 5th Int. Symposium on Large Spatial Databases, (SSD'97)*, pages 298–320. Springer Verlag LNCS-1262, 1997.

[FE92] R.D. Franzosa and M.J. Egenhofer. Topological Spatial Relations Based on Components and Dimensions of Set Intersections. In *Vision Geometry*, SPIE - 1832, pages 236–246. SPIE- The International Society for Optical Engineering, 1992.

[Fra92] A.U. Frank. Qualitative Spatial Reasoning about Distances and Directions in Geographic Space. *Journal of Visual Languages and Computing*, 3:343–371, 1992.

[Fra96] A.U. Frank. Qualitative Spatial Reasoning: Cardinal Directions. *Journal of Geographical Information Systems*, 10(3):269–290, 1996.

[Fre91a] C. Freksa. Conceptual Neighborhood and its Role in Temporal and Spatial Reasoning. In *Decision Support Systems and Qualitative Reasoning*, pages 181–187, 1991.

[Fre91b] C. Freksa. Conceptual neighborhood and its role in temporal and spatial reasoning. In *Decision Support Systems and Qualitative Reasoning*, pages 181–187. Elsevier Science Publishers, (North Holland), 1991.

[Fre92] C. Freksa. Using orientation information for qualitative spatial reasoning. In *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*, LNCS 716, pages 162–178. Springer Verlag, 1992.

[Gal94] A. Galton. Lines of Sight. In *AI and Cognitive Science*, pages 103–113, 1994.

[Gal98] A. Galton. Models of Overlap. *Journal of Visual Languages and Computing*, 9:61–79, 1998.

[Got94a] N.M. Gotts. Defining a 'Doughnut' made Difficult. In *Topological Foundations of Cognitive Science, Column 37 of Reports of the Doctoral Programme in Cognitive Science*. University of Hamburg, 1994.

[Got94b] N.M. Gotts. How far can we 'C'? In *Principles of Knowledge Representation and Reasoning: Proc 4th Intl. Cong. (KR94)*. Morgan Kaufmann, 1994.

[Goy00] R.K. Goyal. *Similarity Assessment for Cardinal Directions Between Extended Spatial Objects*. PhD thesis, University of Maine, 2000.

[GR02] Alfonso Gerevini and Jochen Renz. Combining topological and size information for spatial reasoning. *Artificial Intelligence*, 137(1-2):1–42, 2002.

[Gue89] Guesgen, H.W. Spatial reasoning based on allen's temporal logic. Technical Report TR-89-049, International Computer Science Institute, Berkeley, California, 1989.

[HC01] S.M. Hazarika and A.G. Cohn. Qualitative Self Localisation using a Spatio-Temporal Ontology: A Preliminary Report. In *IJCAI-01 Workshop of Spatial and Temporal Reasoning with an Agents*, 2001.

[HEF95] J.H. Hong, M.J. Egenhofer, and A.U. Frank. On the Robustness of Qualitative Distance and Direction Reasoning. In *Proceedings of Auto Carto 12*, pages 301–310, 1995.

[Her91] D. Hernandez. Relative Representation of Spatial Knowledge: The 2-D Case. In *Cognitive and Linguistic Aspects of Geographic Space*, pages 373–385. Kluwer Academic Publishers, Dordrecht, 1991.

[HT97] Th. Hadzilacos and N. Tryfona. An extended entity-relationship model for geographic applications. In *SIGMOD RECORD*. ACM, 1997.

[IMBM00] A. Isli, L. Museros, Cabedo, T. Barkowsky, and R. Moratz. A Topological Calculus for Cartographic Entities. In *Spatial Cognition II- Integrating Abstract Theories, Empirical Studies, Formal Models and Practical Applications*, pages 225–238, 2000.

[Isl01] Isli, A. and Haarslev, V. and Moller, R. Combining cardinal direction relations and relative orientation relations in qualitative spatial reasoning. Technical Report FBI-HH-M-304/01, Fachbereich Informatik, Universitat Hamburg, 2001.

[JB94] T.Y. Jen and P. Boursier. A Model for Handling Topological Relationships in a 2D Environment. In *Proceedings of the 6th international Symposium on Spatial Data Handling*, volume 1, pages 73–89, 1994.

[LB85] J. Levesque and H.J. Brachman. A Fundamental Trade-off in Knowledge Representation and Reasoning. In R.J. Brachman and H.J. Levesque, editors, *Readings in Knowledge Representation*, pages 41–70. Morgan Kaufmann, 1985.

[LH91] S-Y Lee and F-J Hsu. Picture Algebra for Spatial Reasoning of Iconic Images Represented in 2D C-string. *Pattern Recognition*, 12:425–435, 1991.

[MB83] J. Malik and T.O. Binford. Reasoning in Time and Space. In *Eighth International Joint Conference on AI, IJCAI - 83*, pages 343–345, Germany, 1983.

[ME94] D. Mark and M. Egenhofer. Modeling spatial relations between lines and regions: Combining formal mathematical models and human subjects testing. *Cartography and GIS*, 21(3):195–212, 1994.

[ME02] Ll Museros and M.T. Escrig. Integrating Qualitatively Time and Topology for spatial Reasoning. In *QR 2002- 16th Int. Workshop on Qualitative Reasoning*, 2002.

[MNM+] J.L.V. Mejino, N.F. Noy, M.A. Musen, J.F. Brinkley, and C. Rosse. Representation of Structural Relationships in the Foundational Model of Anatomy. In *Proceedings, AMIA*, pages 973, Publisher =.

[Mul98] P. Muller. A Qualitative Theory of Motion. In *Proc. of the 6th International Conf. Principles of Knowledge Representation and Reasoning*, pages 131–142, 1998.

[NPS97] V.H. Nguyen, C. Parent, and S. Spaccapietra. Complex Regions in Topological Queries. In *Proceeding of the International Conference on Spatial Information Theory COSIT'97*, volume LNCS 1329, pages 175–192. Springer Verlag, 1997.

[Pap94] D. Papadias. *Relation-Based Representation for Spatial Knowledge.* PhD thesis, National Technical University of Athens, 1994.

[PCX87] D.J. Peuquet and Z. Ci-Xiang. An Algorithm to Determine the Directional Relationship between Arbitrarily-Shaped Polygons in the Plane. *Pattern Recognition*, 20(1):65–74, 1987.

[PE88] D.V. Pullar and M.J. Egenhofer. Towards Formal Definition of Topological Relations Among Spatial Objects. In *Proc. 3RD International Symposium on Spatial Data Handling*, pages 225–241, Sydney, Australia, 1988.

[PS94] D. Papadias and T. Sellis. Qualitative representation of spatial knowledge in two-dimensional space. *VLDB*, 3:479–516, 1994.

[PTSE95] D. Papadias, Y. Theodoridis, T. Sellis, and M. Egnehofer. Topological Relations in the World of Minimum Bounding Rectangles,: a Study with R-Trees. *SIGMOD Record*, 24(2):92–103, 1995.

[PWT02] J. Pacheco, M.T. Wscrig, and F. Toledo. Qualitative Spatial Reasoning on Three Dimensional Orientation Point Objects. In *Sistemas Cualtaticos y Diagnosis, Automatisacion del Razonamiento Cualitativo y Aplicaciones*, pages 81–90, 2002.

[RC92] D.A. Randell and A.G. Cohn. Modelling Topological and Metrical Properties in Physical Processes. In *Principles of Representational Reasoning.* Morgan Kaufmann, 1992.

[RCC92a] D. Randell, Z. Cui, and A. Cohn. A Spatial Logic Based on Regions and Connection. In *Principles of Knowledge Representation and Reasoning, KR'92*, pages 165–176, 1992.

[RCC92b] D.A. Randell, A.G. Cohn, and Z. Cui. Computing Transitivity Tables: A Challenge for Automated Theorem Provers. In *Proceedings of CADE 11*, Lecture Notes In Computer Science, 1992.

[Ren01] J. Renz. A Spatial Odyssey of the Interval Algebra:1. Directed Intervals. In *Proc. of the 17th International Joint Conf. on AI, IJCAI'01*, 2001.

[RS88] G. Retz-Schmidt. Various views on spatial prepositions. *AI Magazine*, 9(2):95–105, 1988.

[RS01] a. Roy and J. Stell. Spatial Relations between Indeterminate Regions. *Journal of Approximate Reasoning*, 27(3):205–234, 2001.

[RW01] D. Randell and M. Wikowski, Shanhan. From Images to Bodies: Modeling and Exploiting Spatial Occlusion and Motion Parallax. In *Proceedings IJCAI 2001*, pages 57–63, 2001.

[RW02] D. Randell and M. Wikowski. Building Large Composition Tables via Axiomatic Theories. In *Principles of Knowledge Representation and Reasoning: Proceedings of the Eighth International Conference (KR-2002)*, pages 26–35. AAAI Press, 2002.

[Sch01] Markus Schneider. A design of topological predicates for complex crisp and fuzzy regions. *Lecture Notes in Computer Science*, 2224:103–??, 2001.

[SK01] S. Skiadopoulos and M. Koubarakis. Composing Cardinal Direction Relations. In *SSTD 2001, LNCS 2121*, pages 299–317, 2001.

[Ste00] J.G. Stell. Boolean connection algebras: A new approach to the region connection calculus. *Artificial Intelligence*, 122(1-2):111–136, 2000.

[TJ] Eddie Tunstel and Mo Jamshidi. Fuzzy relational representation of uncertain spatial maps for autonomous vehicles.

[TN] Erlend Tssebro and Mads Nygrd. An advanced discrete model for uncertain spatial data.

[Tra98] A. Traverse. *Interval-Based Qualitative Spatial Reasoning*. PhD thesis, Curtin University of Technology, 1998.

[Tve93] B. Tversky. Cognitive Maps, Cognitive Collages and Spatial Mental Models. In *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*, pages 14–24, 1993.

[Vaz00] M. Vazirgiannis. Uncertainty Handling in Spatial Relationships. In *Proc. of the 2000 ACM Symposium on Applied Computing*, pages 494–500, 2000.

[Vie93] L. Vieu. A Logical Framework for Reasoning about Space. In *Spatial Information Theory, A Theoretical Basis for GIS : European Conference, COSIT'93*, LNCS 716, pages 25–35. Springer Verlag, September 1993.

[WC01] M.F. Worboys and E. Clementini. Integration of imperfect spatial information. *Journal of Visual Languages and Computing*, 12:61–80, 2001.

[Zha98] F.B. Zhan. Approximate analysis of binary topological relations between geographic regions with indeterminate boundaries. *Soft Computing*, 2:28–34, 1998.

[Zim92] K. Zimmermann. A Proposal for Representing Object Sizes. In *ECAI 92 Workshop notes, Spatial Concepts: Connecting Cognitive Theories with Formal Representation*, 1992.

[Zim93] K. Zimmermann. Enhancing Qualitative Spatial Reasoning - Combining Orientation and Distance. In *Spatial Information Theory, A Theoretical Basis for GIS : European Conference, COSIT'93*, LNCS 716, pages 69–76. Springer Verlag, September 1993.